AdS/CFTのコライダー物理への応用

八田佳孝

（筑波大数理）

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Why AdS/CFT?

- Perturbative QCD very successful for hard processes. Why bother AdS?
- High CM energy does not mean weak coupling. There are many unsolved problems
- Hope AdS/CFT can bring new insights into the soft aspects of high energy QCD. Certain features may be universal.

Collider experiments

LEP (CERN,1989~2000)

electron-positron annihilation $\sqrt{s} < 200\mathrm{GeV}$

HERA (DESY, 1990~2007)

Deep inelastic scattering (DIS) $\sqrt{s} \approx 320\mathrm{GeV}$

Tevatron (Fermilab, 1987~) proton-antiproton $\sqrt{s} = 1.96\ \mathrm{TeV}$

RHIC (Brookhaven, 2000~) proton-proton
nucleus-nucleus, $\sqrt{s_{NN}} \approx 200\ \mathrm{GeV}$

LHC (CERN, 2008~) proton-proton, $\sqrt{s} = 900\ \mathrm{GeV}, 2.36\ \mathrm{TeV},\ \ 7\ \mathrm{TeV}, \ldots$
nucleus-nucleus $\sqrt{s_{NN}} \approx 5.5\mathrm{TeV}$

ILC (??, 20??) electron—positron annihilation $\sqrt{s} > 500\ \mathrm{GeV}$

Processes studied so far

- Hard scattering Polchinski-Strassler
- DIS and Pomeron

  Janik-Peschanski; Polchinski-Strassler; Brower-Polchinski-Strassler-Tan
  Cornalba-Costa-Penedones; BallonBayona-BoschiFilho-Braga;
  AlvarezGaume-Gomez-VasquezMozo; YH-Iancu-Mueller…
- e+e- annihilation

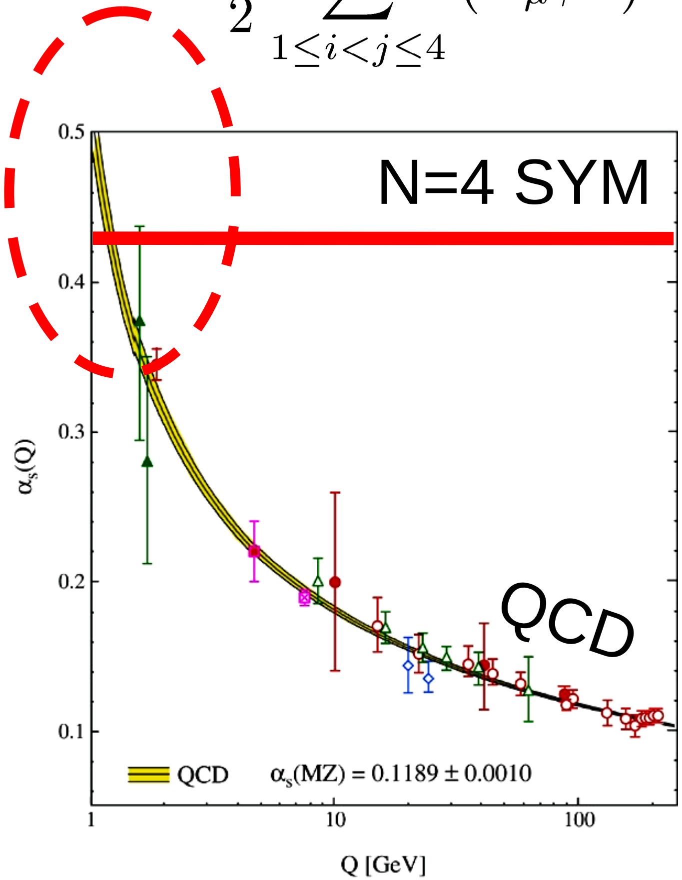
  Hofman-Maldacena; YH-Iancu-Mueller; Evans-Tedder
  Chesler-Jensen-Karch; YH-Matsuo; Csaki-Reece-Terning…
- Nucleus-nucleus collision

  Shuryak-Lin, Nastase, Albacete-Kovchegov-Taliotis; Gubser-Pufu-Yarom…
- Polarized DIS Gao-Xiao, YH-Ueda-Xiao…
- Odderon Brower-Djuric-Tan, Avsar-YH-Matsuo
- Drell-Yan BallonBayona-BoschiFilho-Braga

N=4 supersymmetric Yang-Mills

$$\mathcal{L} \; = \; -\frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu}_{a} + \sum_{i=1}^{4}\bar{\psi}^{i}_{\alpha}(\bar{\sigma}\cdot D)_{\alpha\beta}\psi^{i}_{\beta} + 2\sqrt{2}g f^{abc}\sum_{1\leq i<j\leq 4} Re\left(\phi^{ij}_{a}\psi^{i\alpha}_{b}\psi^{j}_{c\alpha}\right)$$

$$+\frac{1}{2}\sum_{1\leq i<j\leq 4}\left(D_{\mu}\phi^{ij}\right)^{\dagger}D^{\mu}\phi^{ij} - \frac{g^{2}}{4}\sum_{\substack{1\leq i<j\leq 4\\ 1\leq k<l\leq 4}}|f_{abc}\phi^{ij}_{b}\phi^{kl}_{c}|^{2}$$



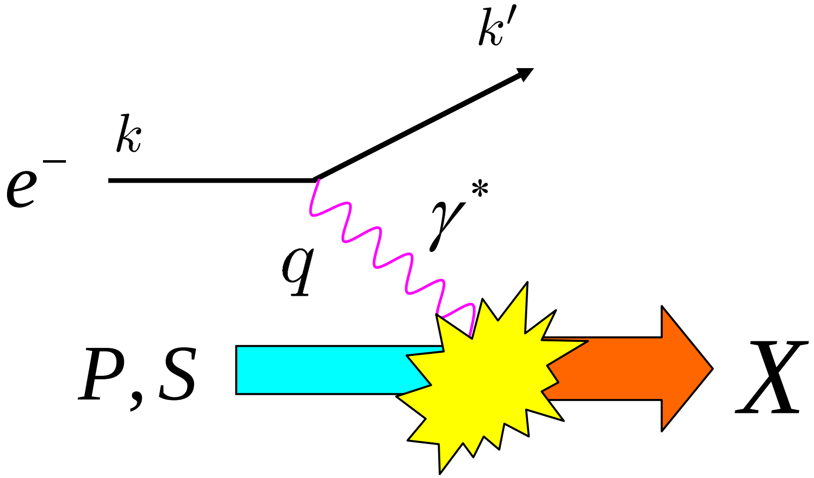
Gauge boson (“gluon”),
4 Weyl fermions (“quarks”),
6 scalars, all in the adjoint rep.

Global SU(4) R-symmetry

Dual to type-IIB superstring on

$$AdS_5 \times S^5$$

Deep inelastic scattering



Photon virtuality

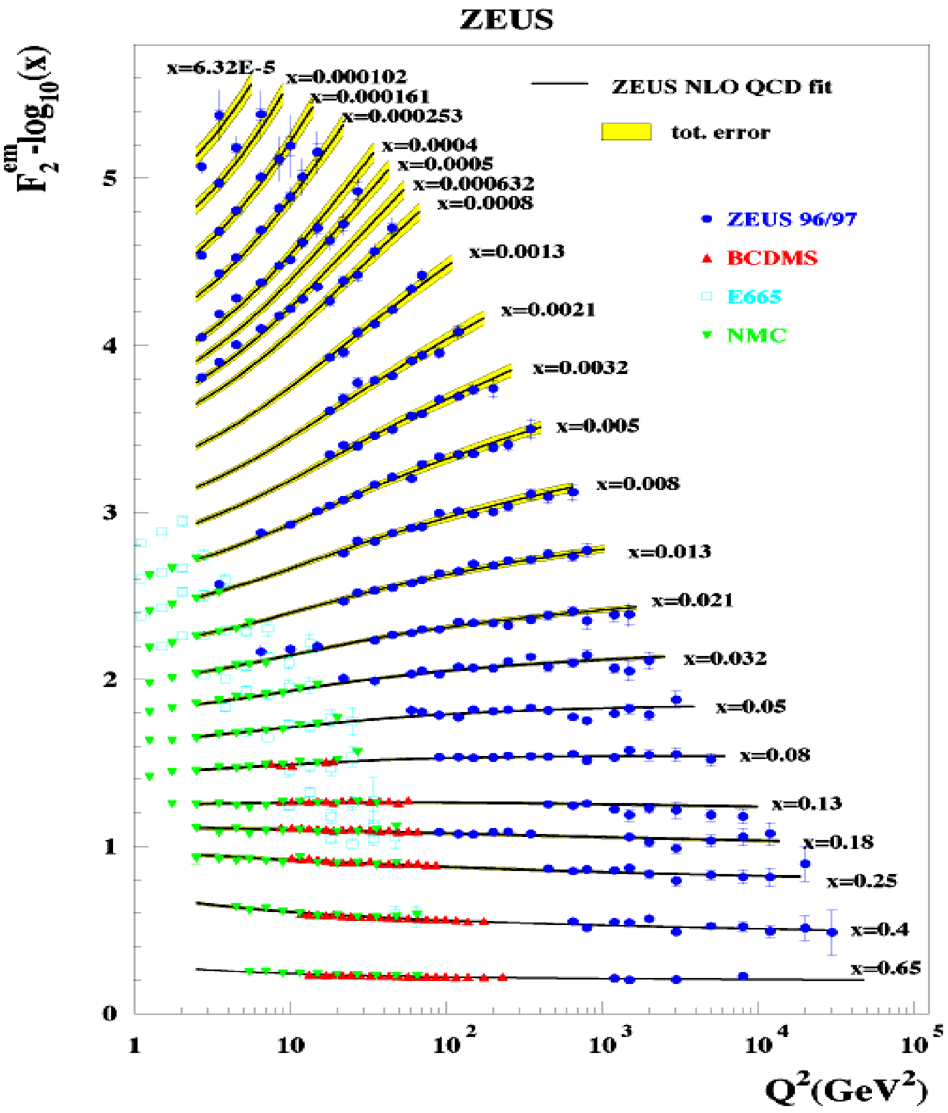
$$q^2 = -Q^2 < 0 \qquad \text{(spacelike)}$$

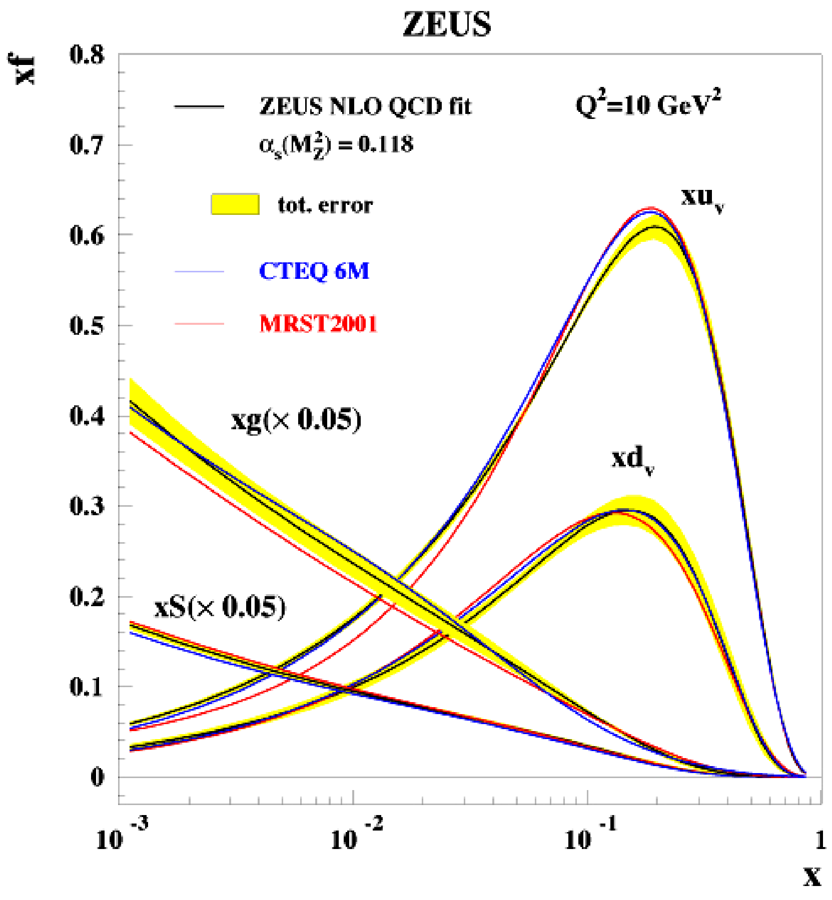
Bjorken variable

$$\begin{aligned} x &= \frac{Q^2}{2P\cdot q} \\ &\approx \frac{Q^2}{s} \quad (x \ll 1) \end{aligned}$$

$$\begin{aligned} &\frac{1}{4\pi}\int d^4y e^{iqy}\langle P,S|[J^\mu(y),J^\nu(0)]|P,S\rangle \\ &= \left(-\eta^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{P\cdot q} \\ &\quad + i\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(\frac{S_\beta}{P\cdot q}(g_1(x,Q^2) + g_2(x.Q^2)) - \frac{q\cdot SP_\beta}{(P\cdot q)^2}g_2(x,Q^2)\right) \end{aligned}$$

Parton evolution





Hard Pomeron

$$\int_0^1 dx x^{j-2} F_2(x, Q^2) \sim \left(\frac{Q^2}{\mu^2}\right)^{\gamma(j)}$$

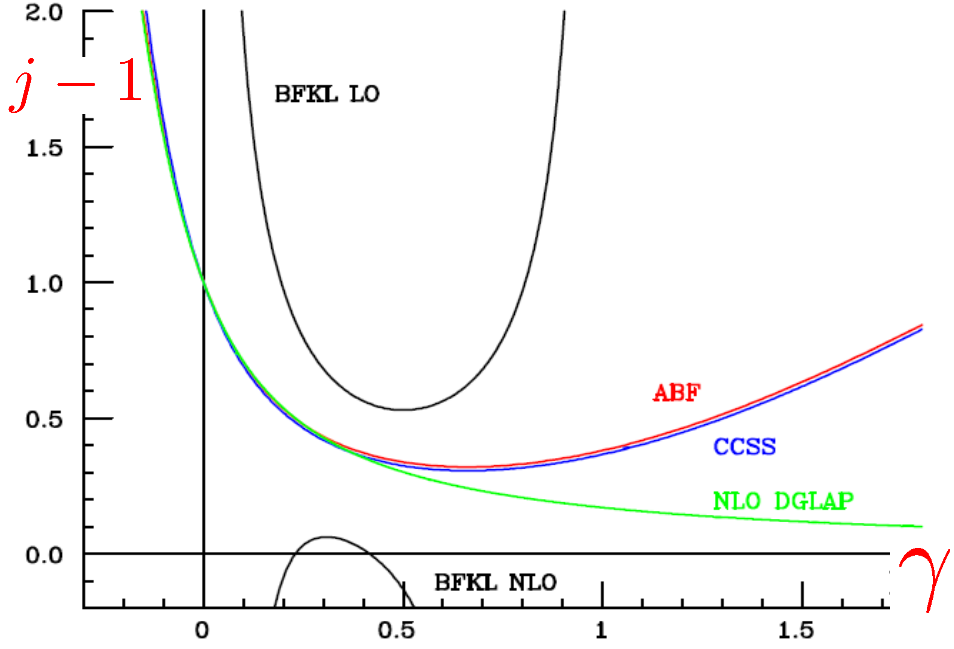
Anomalous dim. of the
twist-two operators

$$\bar{q}_f(0)\gamma^+(iD^+)^{j-1}q_f(0), \ldots$$

$$F_2(x, Q^2)$$

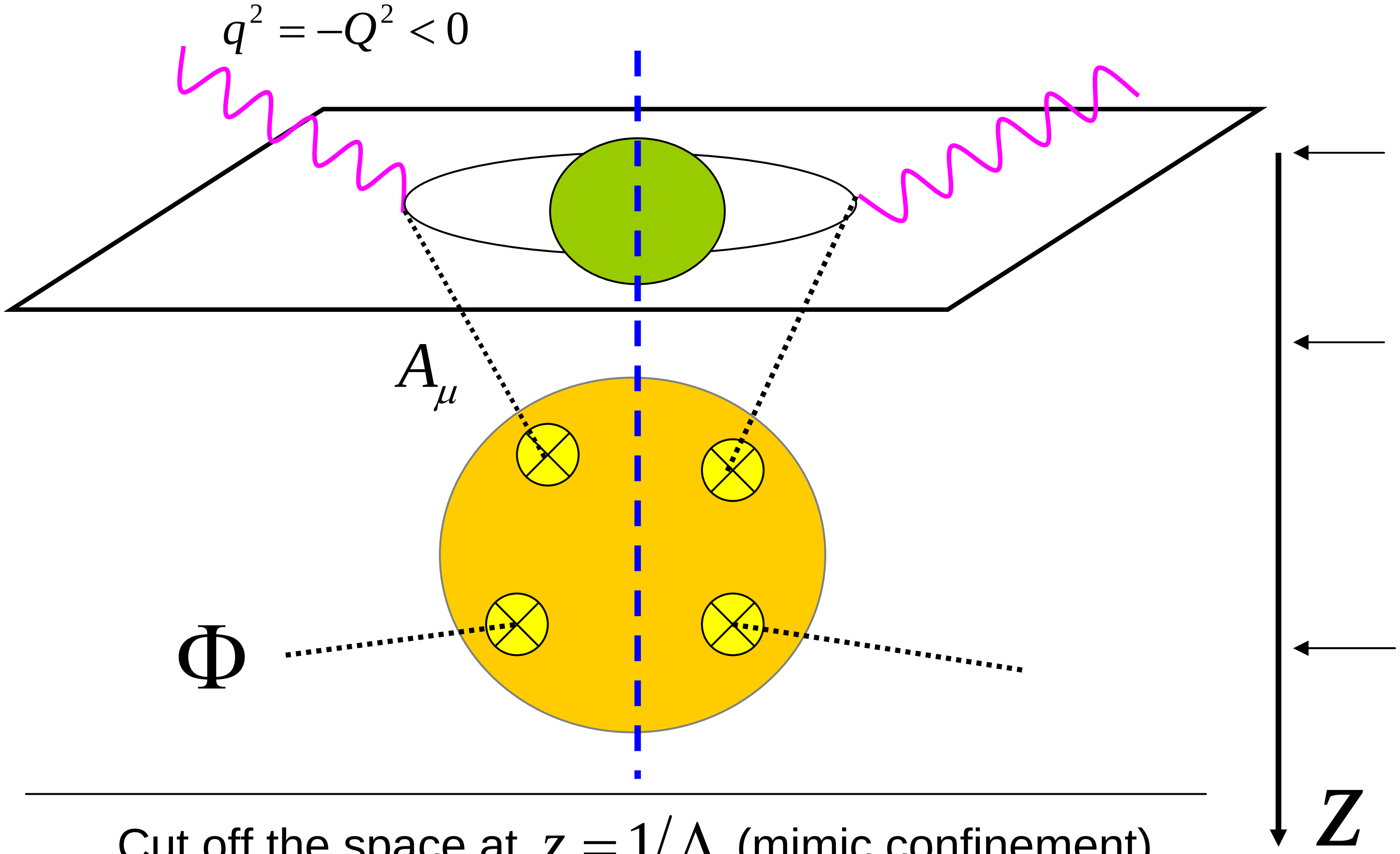
$$\sim \int \frac{dj}{2\pi i}\left(\frac{Q^2}{\mu^2}\right)^{\gamma(j)}\left(\frac{1}{x}\right)^{j-1}$$

Analytically continued
to non-integer j



DIS at strong coupling

Polchinski-Strassler (2002)
YH-Iancu-Mueller (2007)



Our universe
$(z = 0)$

Photon localized
at small $z \propto 1/Q$

Target localized
at large $z \sim 1/\Lambda$

Cut off the space at $z = 1/\Lambda$ (mimic confinement)

Large-x : No partons !

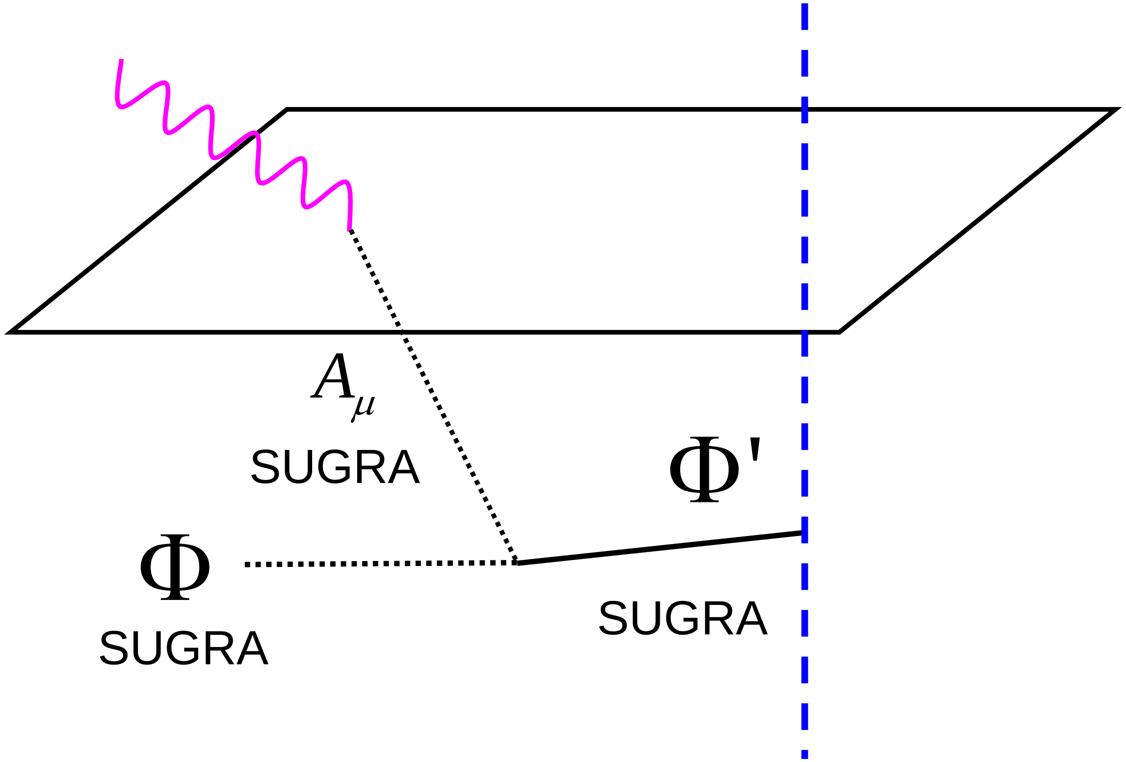
Polchinski, Strassler (2002)

At large-x, a hadron scatters as a whole.

$$s_{4D} = \frac{Q^2}{x}$$

$$s_{5D} = \frac{R^2}{r_{int}^2} s_{4D} < \frac{1}{\alpha'}$$

$$\Rightarrow \quad \frac{1}{\sqrt{\lambda}} < x < 1$$

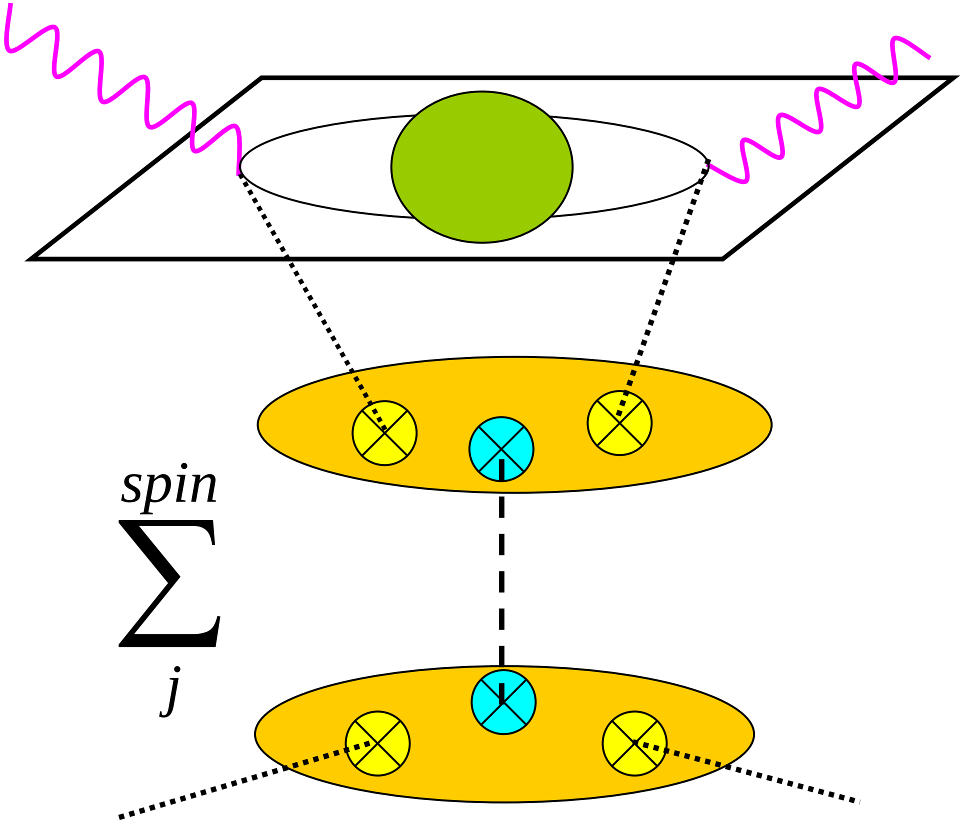


So different from QCD.
Phenomenology? No way !

Small-x: Regge regime

Brower, Polchinski, Strassler, Tan (2006)

$$A \sim s^{2+\frac{\alpha' t}{2}}$$
$$= \int \frac{dj}{2\pi i} \frac{s^j}{j - 2 - t/2\sqrt{\lambda}}$$



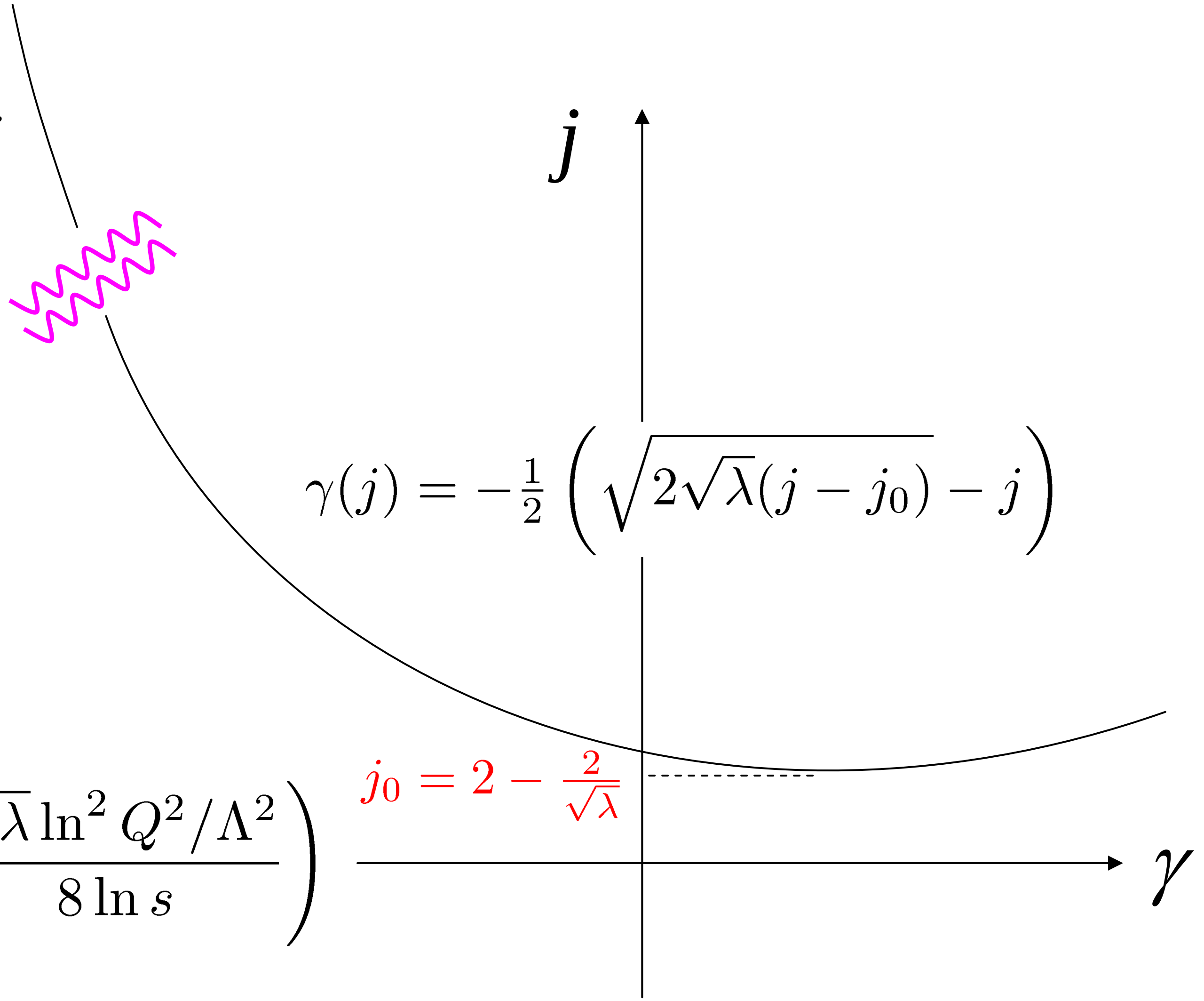
$$t_{5D} = t_\perp + t_z$$

Laplacian operator acting on
the string exchanged in the t-channel

Eigenvalue of t_z shifts the intercept $j_0 = 2 + \mathcal{O}(1/\sqrt{\lambda})$

Pomeron= Reggeized graviton in AdS

The anomalous dimension at strong coupling



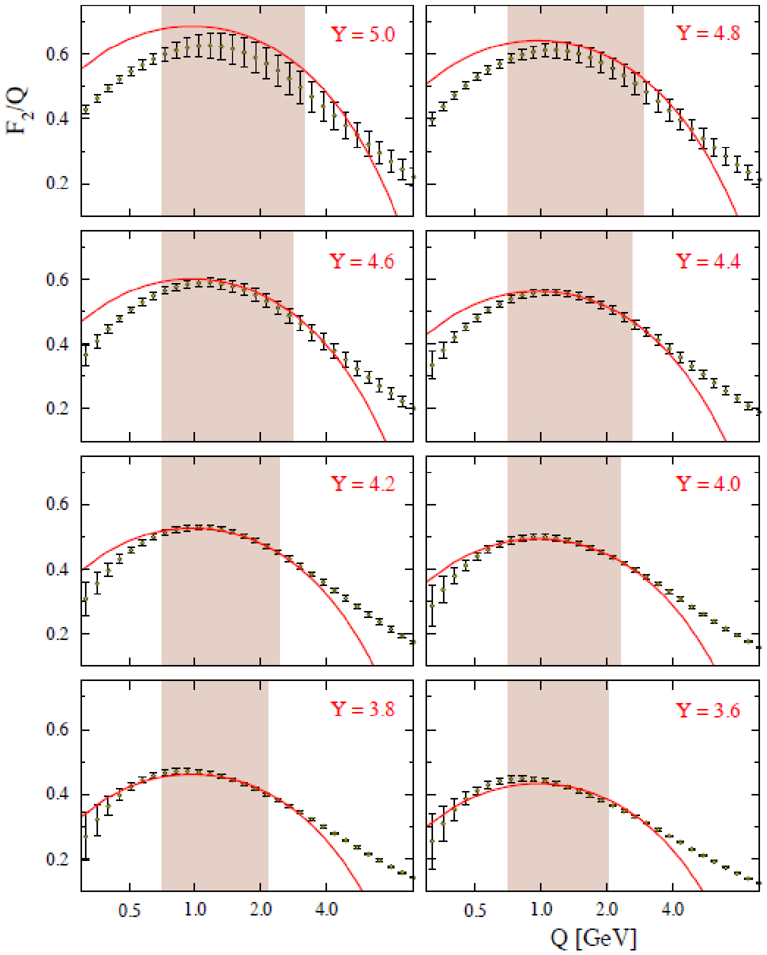
$$\gamma(j) \approx -\frac{1}{2}\frac{\sqrt{\lambda}}{\pi}\ln j$$

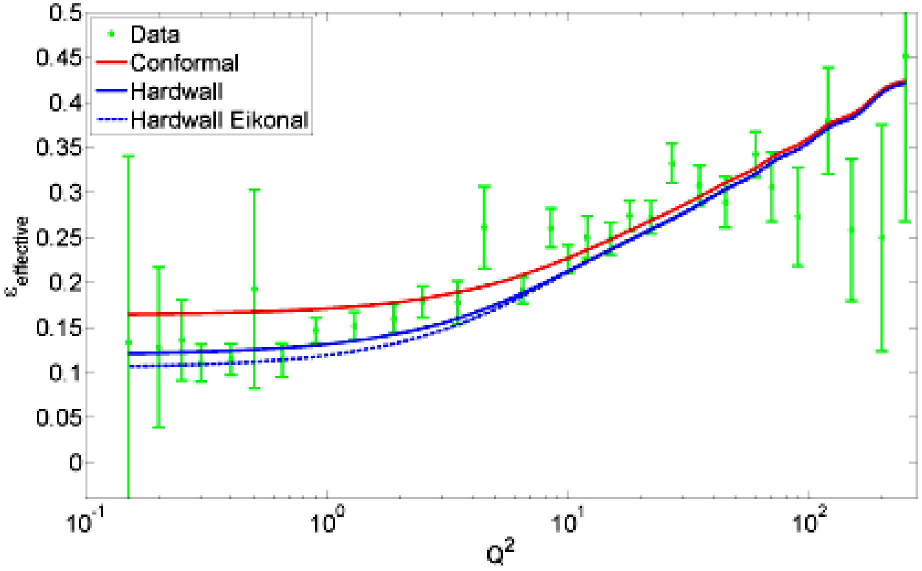
$$\gamma(j) = -\frac{1}{2}\left(\sqrt{2\sqrt{\lambda}(j-j_0)}-j\right)$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

$$F_2(x,Q^2) \sim \frac{s^{1-2/\sqrt{\lambda}}}{N_c^2}\exp\left(-\frac{\sqrt{\lambda}\ln^2 Q^2/\Lambda^2}{8\ln s}\right)$$

Fits to the HERA data





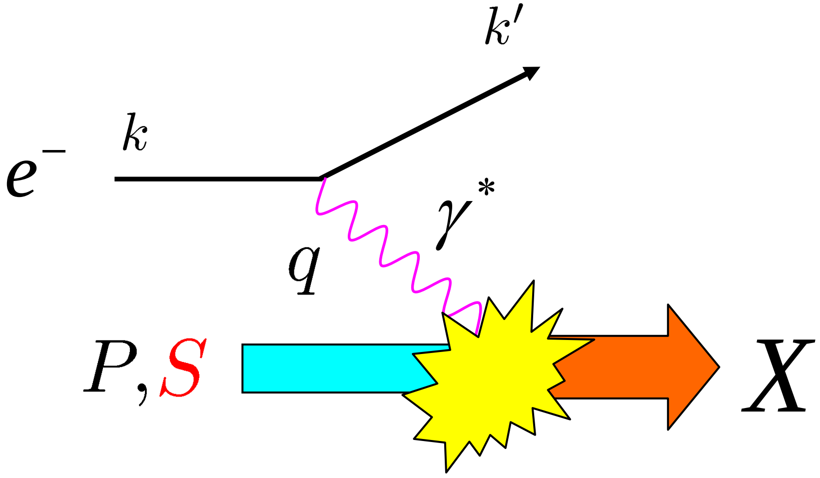
$$F_2(x,Q^2) \sim \left(\frac{1}{x}\right)^{\epsilon_{eff}(Q^2)}$$

Cornalba & Costa; 0804.1562

Brower, Djuric, Sarcevic & Tan; 1007.2259

Polarized DIS

DIS on a longitudinally polarized proton



$$\frac{1}{4\pi}\int d^4y e^{iqy}\langle P,S|[J^\mu(y),J^\nu(0)]|P,S\rangle$$

$$\sim i\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(\frac{S_\beta}{P\cdot q}(g_1(x,Q^2)+g_2(x.Q^2))-\frac{q\cdot SP_\beta}{(P\cdot q)^2}g_2(x,Q^2)\right)$$

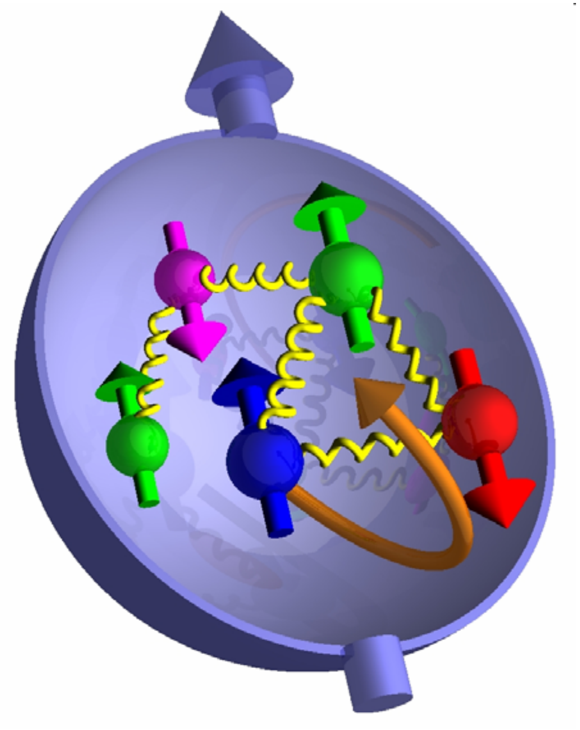
$$\int_0^1 dx\, g_1(x,Q^2)=\frac{1}{4S^+}\sum_f e_f^2\langle PS|\bar{q}_f\gamma_5\gamma^+q_f|PS\rangle=\frac{1}{9}\Delta\Sigma+\frac{1}{12}a^{(3)}+\frac{1}{36}a^{(8)}$$

↑

Quarks’ helicity contribution to the proton spin

$\Delta\Sigma = 1$ in the naïve quark model

Spin crisis



Angular momentum decomposition in QCD

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z$$

Quarks’ helicity

Gluons’ helicity

Oribital angular momentum

The EMC shock (1987)

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \quad !?$$

Polarized DIS from AdS/CFT

YH, Ueda, Xiao (2009)

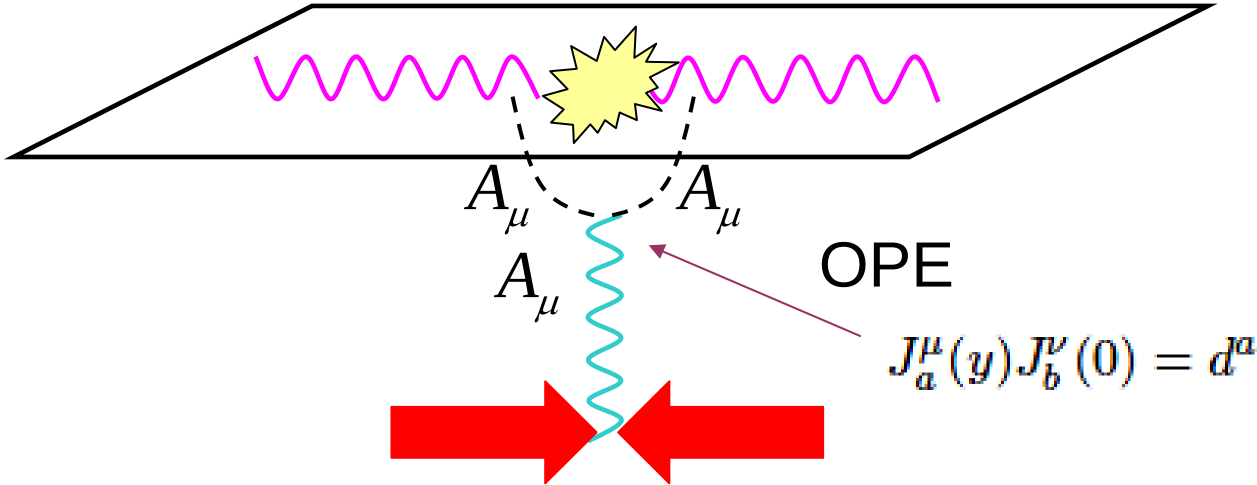
Unpolarized
→ graviton exchange j=2

$$JJ \sim \frac{1}{x^2}T \qquad F_1(x,Q^2) \sim \left(\frac{1}{x}\right)^{2-\mathcal{O}(1/\sqrt{\lambda})}$$

質問にちゃんと答えられませんでしたが、この仕事では
N=4 SYMのフェルミオンをクォークの代用物として扱っています。

Polarized
→ SO(6) gauge boson exchange j=1

$$JJ \sim \frac{1}{x}J \qquad g_1(x,Q^2),\ g_2(x,Q^2) \sim \left(\frac{1}{x}\right)^{1-\mathcal{O}(1/\sqrt{\lambda})}$$



$$J_a^\mu(y)J_b^\nu(0) = d^{abc}\epsilon^{\mu\nu}{}_{\alpha\beta}\frac{y^\alpha}{3\pi^2 y^4}J_c^\beta(0) + \cdots$$

g1 and g2 at strong coupling

$$g_1(x,Q^2)S^+ + \frac{F_3(x,Q^2)}{2}P^+ = \frac{d^{33c}Q_c}{6\pi}\frac{\pi}{4\sqrt{\lambda}}\int d^4y dz\sqrt{G}\int dz'\sqrt{G'}G'^{+-}zz'$$

$$\times\left(\frac{1}{x}\right)^{1-\frac{1}{2\sqrt{\lambda}}}\frac{e^{-(\rho-\rho')^2/4D\tau}}{\sqrt{\pi D\tau}}J_+^{bulk}(z',y)\bar{\psi}\gamma^+\psi(z)$$

$$(\tau = \ln 1/x)$$

Sum rule OK.

The g2 structure function is much smaller than g1.

$$g_1(x,Q^2) \sim \frac{c}{x^{1-\epsilon}} \qquad \epsilon \sim \mathcal{O}(1/\sqrt{\lambda})$$

$$\longrightarrow \quad g_2(x) \sim -\frac{c}{1-\epsilon} + \frac{\epsilon c}{(1-\epsilon)x^{1-\epsilon}}$$

Wandzura-Wilczek relation

Comparison with QCD

QCD

N=4 SYM at strong coupling

Experiment

$$\Delta\Sigma \sim 0.3$$

$$\Delta G \approx 0.0 \pm 0.5$$

$$\Delta\Sigma \approx 0$$

$$\Delta G \approx 0\ ??$$

$$g_1^{singlet}(x) \approx 0$$

$$g_1^{non-singlet}(x) \sim \left(\frac{1}{x}\right)^{0.5} ??$$

Similar

$$g_1^{singlet}(x) \approx 0$$

$$g_1^{non-singlet}(x) \sim \left(\frac{1}{x}\right)^{1-1/2\sqrt{\lambda}}$$

Opposite

pQCD

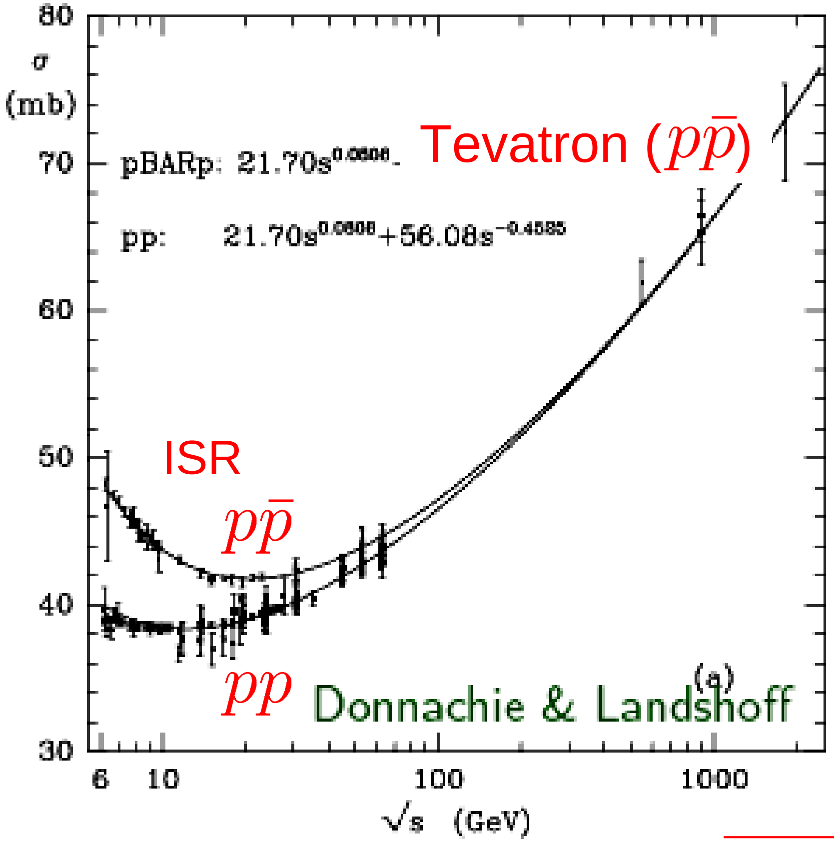
$$g_1^{singlet}(x) \sim \left(\frac{1}{x}\right)^{2.5\frac{\sqrt{\lambda}}{2\pi}}$$

$$g_1^{non-singlet}(x) \sim \left(\frac{1}{x}\right)^{\frac{\sqrt{\lambda}}{2\pi}}$$

Helicity contribution suppressed due to the large anomalous dimension.

Total cross section difference

Consider the difference $\Delta\sigma \equiv \sigma_{tot}^{p\bar{p}} - \sigma_{tot}^{pp}$



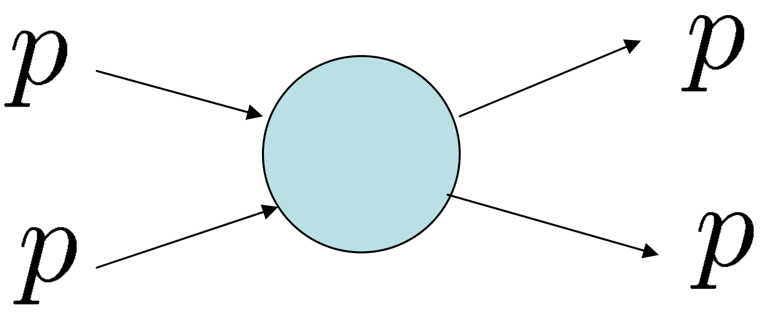
Up to the ISR energy, $(\sqrt{s} = 53\mathrm{GeV})$

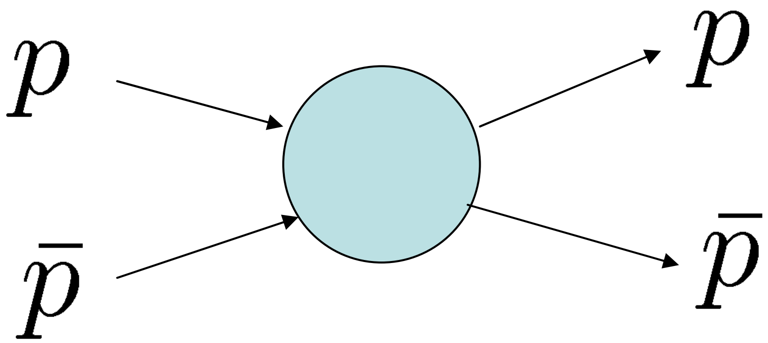
$$\Delta\sigma \sim s^{-0.5} > 0$$

What's the fate of $\Delta\sigma$ as $s \to \infty$??

LHC(pp), cosmic rays

Odderon and Reggeon





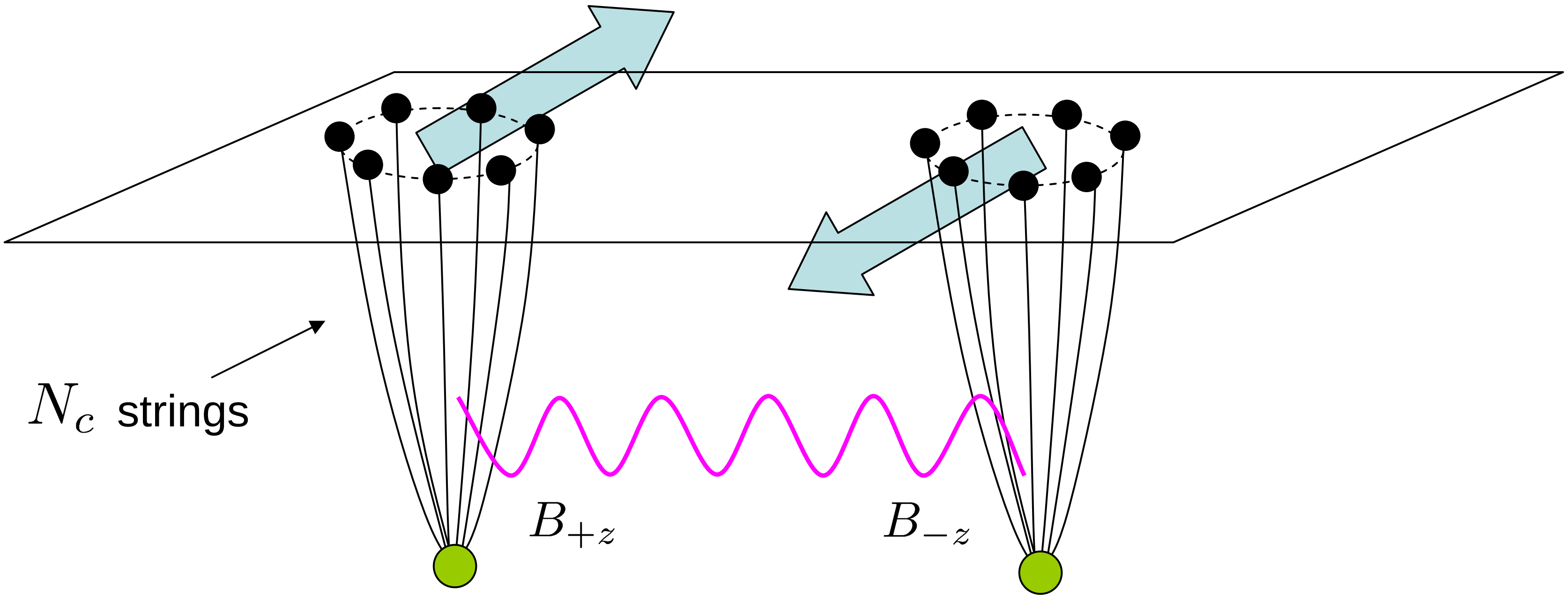
The difference $\Delta\mathcal{A} \equiv \mathcal{A}_{pp\to pp}(s,t) - \mathcal{A}_{p\bar{p}\to p\bar{p}}(s,t)$

is odd under crossing, generated by the exchange of C-odd objects in QCD

$\longrightarrow \left\{ \begin{array}{l} \text{Reggeon (vector mesons)} \\ \text{Odderon (C-odd glueballs)} \end{array} \right.$ Lukaszuk, Nicolescu (1973)

$$\Delta\sigma(s) = \frac{1}{s}\,\mathrm{Im}\Delta\mathcal{A}(s,t=0) \sim s^{-0.5}$$

Odderon = antisymmetric B-field



Baryons as wrapped D-branes

Witten (1998)
Imamura (1999)

B-field couples to the electric field in the D-brane worldvolume

Opposite sign in pp and ppbar scattering.

Total cross section difference

Avsar, YH, Matsuo (2009)

$$\Delta\sigma = \sigma^{BB} - \sigma^{BB} = 2\int d^2b\, Im\, \mathcal{A}^-(s,b) - 2\int d^2b\, Im\, \mathcal{A}^+(s,b)$$

$$= -\frac{\pi\sqrt{\lambda}}{4(\mathrm{Vol}_{S^4})^2}\sum_{I,k}\frac{M_I + \frac{1}{M_I}}{k+2}\int dz d\Omega_4 Y^{(k)}(\Omega_5)\int dz' d\Omega_4' Y^{(k)}(\Omega_5')\left(\frac{zz's}{4\sqrt{\lambda}}\right)^{\alpha_O(0)-1}.$$

$$\alpha_O(0) = 1 - \frac{M^2-1}{2\sqrt{\lambda}} \qquad M = 1, 2, \ldots$$

$\Delta\sigma$ is negative ! …in conflict with the ISR data

sign of $\Delta\sigma$ → sign of $\mathrm{Im}\mathcal{A}$ → sign of the interaction

exchange of the B-field → repulsion → $\Delta\sigma > 0$??

However, this may not be true in a curved space !!

A prediction

Reggeon exchange gives a positive contribution

$$\Delta\sigma \sim s^{\alpha_R - 1} > 0 \qquad \alpha_R = 1 - \frac{9}{2\sqrt{\lambda}},\quad 1 - \frac{16}{2\sqrt{\lambda}}, \cdots$$

Odderon exchange gives a negative contribution

$$\Delta\sigma \sim s^{\alpha_O - 1} < 0 \qquad \alpha_O = 1,\quad 1 - \frac{3}{2\sqrt{\lambda}},\quad 1 - \frac{8}{2\sqrt{\lambda}}, \cdots$$

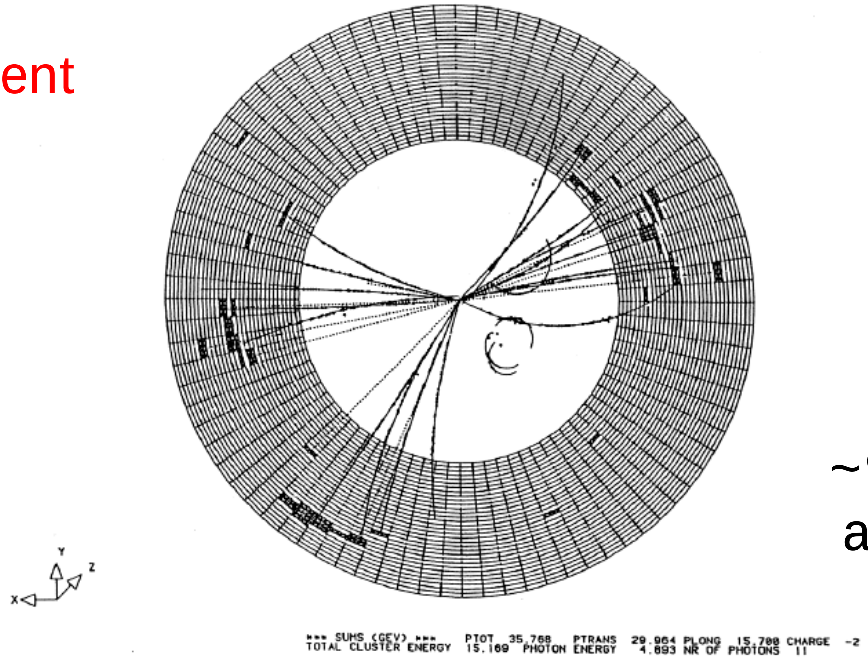
At the ISR energies, the Reggeon dominates.
But the Odderon eventually takes over, possibly at the LHC!

オデロンが見えてくるのは $\ln s > \sqrt{\lambda}$ の高いエネルギー領域であり、LHCではこの条件は満たされていると考えられます。

Jets

Observation of jets in 1975 has provided one of the most striking confirmations of QCD

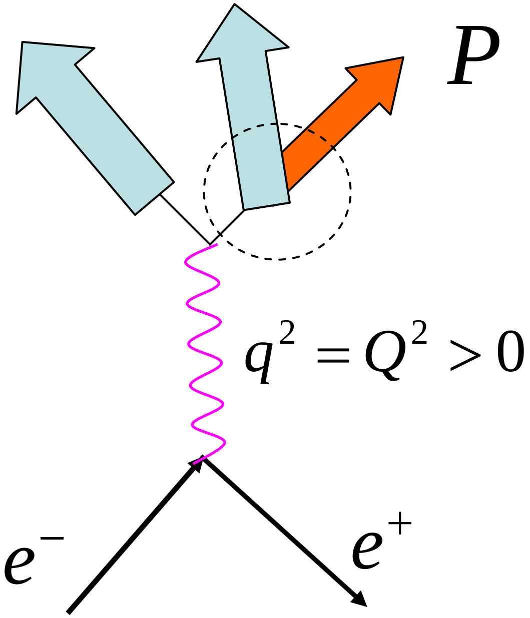
A three-jet event



~90% of the final states are two-jet events

Average angular distribution $1+\cos^2\theta$ reflecting fermionic degrees of freedom

Fragmentation function



Feynman-x $\quad x \equiv \frac{2P\cdot q}{Q^2} = \frac{2E}{Q}$

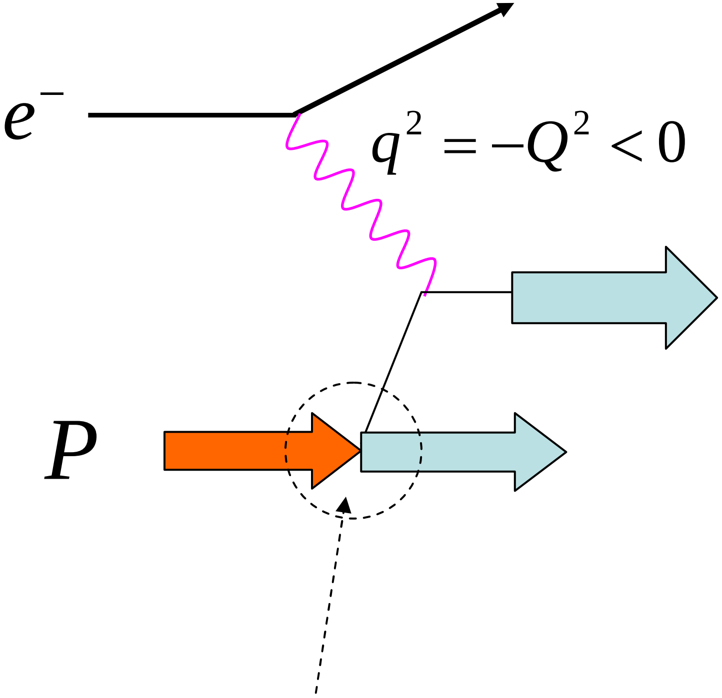
$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx} \sim D_T(x,Q^2)$$

Count how many hadrons are there inside a quark.

$$\frac{\partial}{\partial \ln Q^2} D_T(j,Q^2) = \gamma_T(j) D_T(j,Q^2)$$

Timelike anomalous dimension

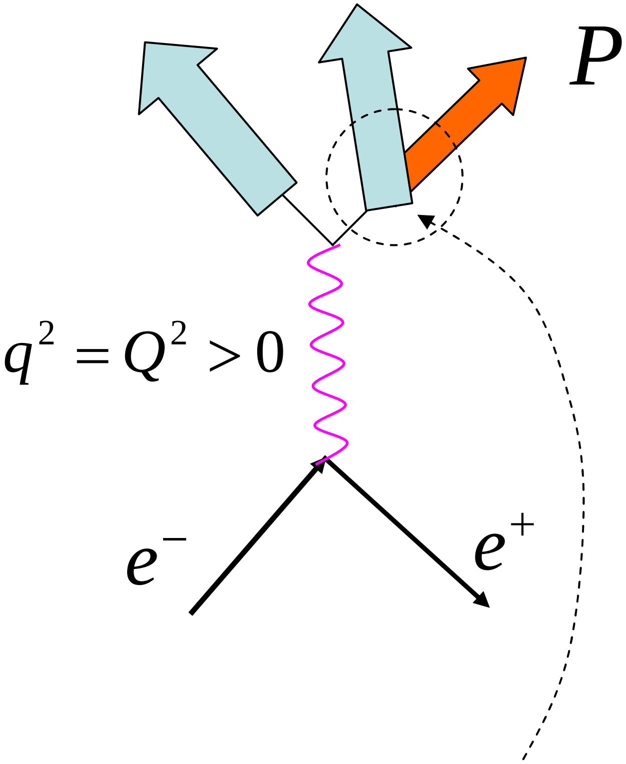
DIS vs. e+e- : crossing symmetry



Parton distribution function

$D_S(x_B, Q^2)$

Bjorken variable $x_B \equiv \frac{Q^2}{2P \cdot q}$



Fragmentation function

$D_T(x_F, Q^2)$

Feynman variable $x_F \equiv \frac{2P \cdot q}{Q^2}$

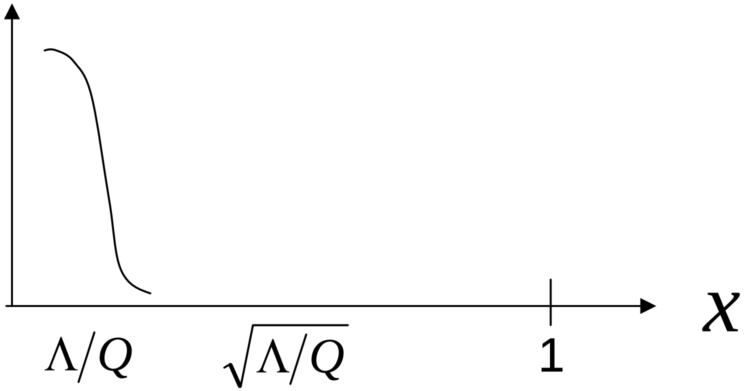
Fragmentation function at strong coupling

YH, Matsuo (2008)

$$\gamma_S(j) = \frac{j}{2} - \frac{1}{2}\sqrt{2\sqrt{\lambda}(j - j_0)} \quad \xleftrightarrow{\text{crossing}} \quad \gamma_T(j) = -\frac{1}{2}\left(j - j_0 - \frac{j^2}{2\sqrt{\lambda}} \right)$$

$$n(Q) \propto (Q/\Lambda)^{2\gamma_T(1)} = (Q/\Lambda)^{1-3/2\sqrt{\lambda}}$$

$$D_T(x, Q^2) = Q^2 F\left(\frac{Qx}{\Lambda}\right)$$



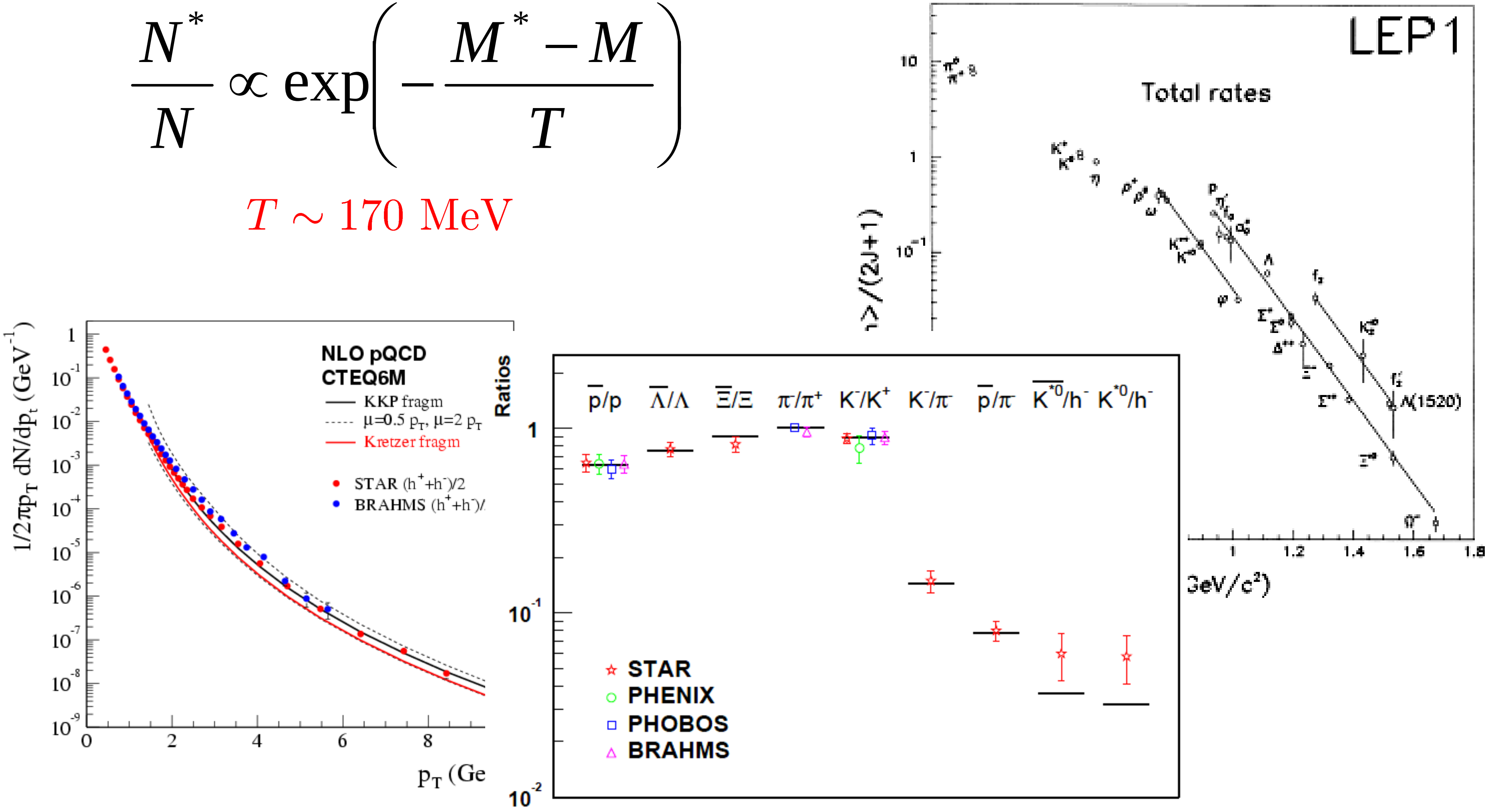
At strong coupling, branching is so fast and complete.
Nothing remains at large-x !

Thermal hadron production

Identified particle yields are well described by a thermal model

$$\frac{N^*}{N} \propto \exp\left(-\frac{M^*-M}{T}\right)$$

$T \sim 170$ MeV



Thermal hadron production from gauge/string duality

string amplitude

$$\langle 0|\epsilon \cdot j(0)|p_1, ...p_n\rangle$$
$$\sim \frac{g_c^{n+1}}{\alpha' g_c^2} \int dz d\Omega_5 \sqrt{-G} F(\alpha' \partial^2)(\Phi)^n A_\mu$$

5D hadron w.f.

5D photon

Polchinski, Strassler (2001)

When $n \sim \mathcal{O}(1)$ amplitude dominated by $z_s \sim \frac{1}{p}$

$\longrightarrow$ Dimensional counting rule by Brodsky, Farrar (1973)

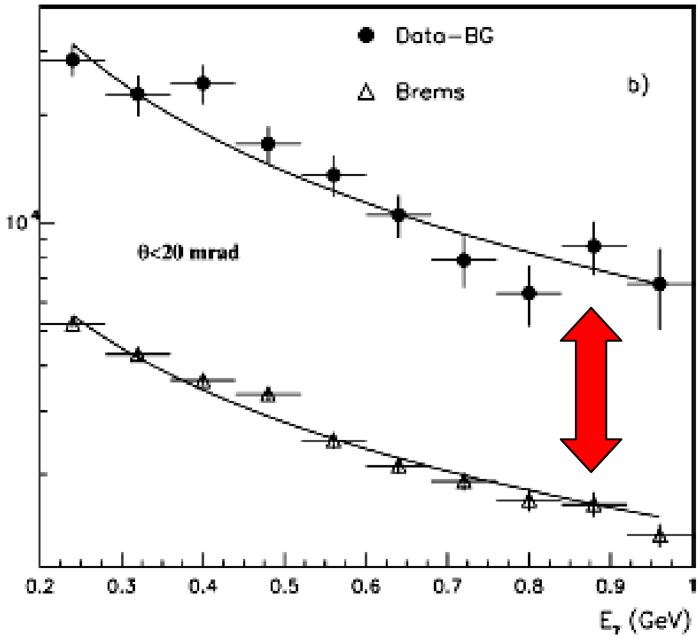
YH, Matsuo (2008)

When $n \sim \mathcal{O}(Q)$ and $A_\mu \propto H_1^{(1)}(Qz) \sim e^{iQz}$

Saddle point imaginary $z_s \sim \frac{i}{\Lambda}$ $\quad e^{iQz_s} \sim e^{-Q/\Lambda}$

Thermal !

Soft photon puzzle



YH, Ueda (2010)
T.Ueda (talk yesterday)

Factor 5 discrepancy between the data and theory (Bremsstrahlung).

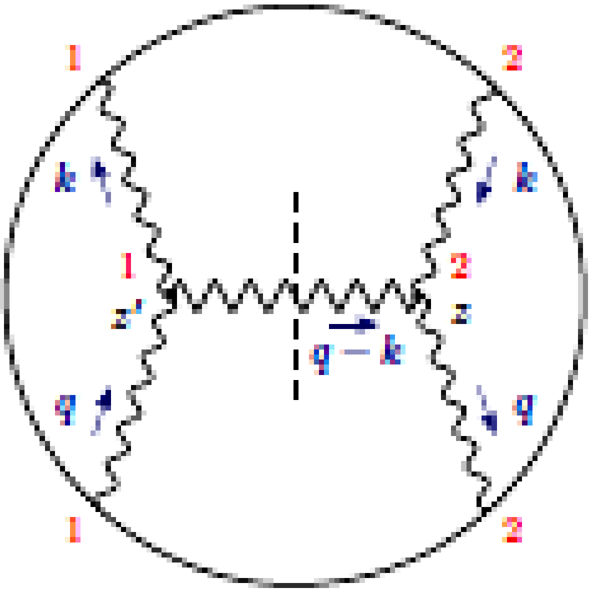
Most recent analysis by DELPHI (e+e-)

Compute the four-point correlator of the R-current operator from AdS/CFT

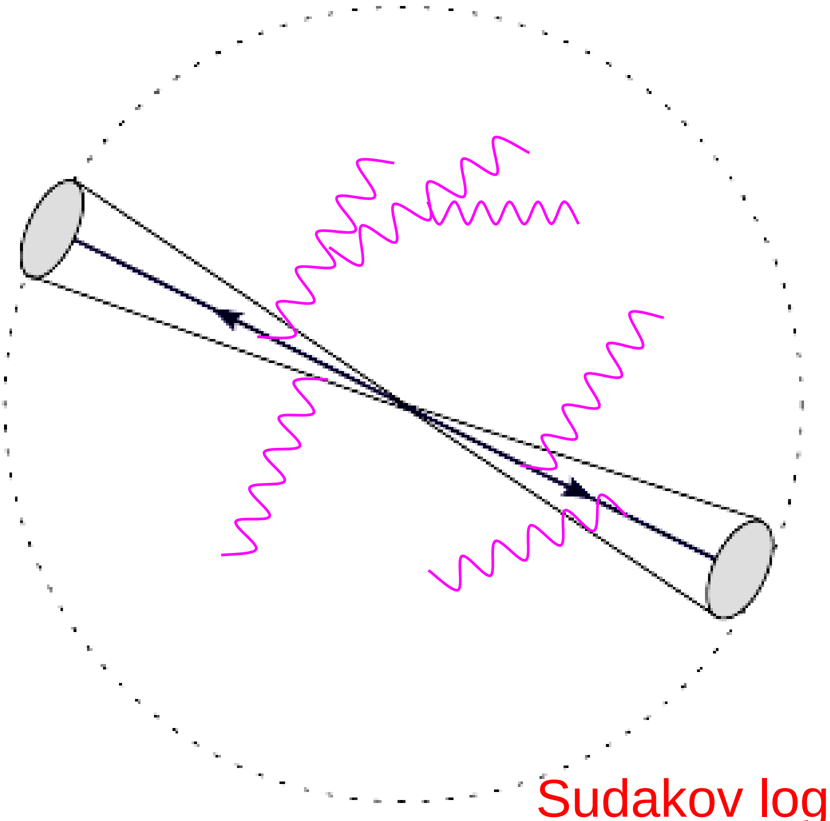
$$\frac{dN}{dk} = \frac{\alpha_{em}}{2\pi k}$$

Exact !

Novel source of soft photons



Away-from-jets region



Gluons emitted at large angle, insensitive to the collinear singularity

Resum only the soft logarithms

$$\left(\alpha_s \ln 1/x\right)^n$$

There are two types of logarithms.

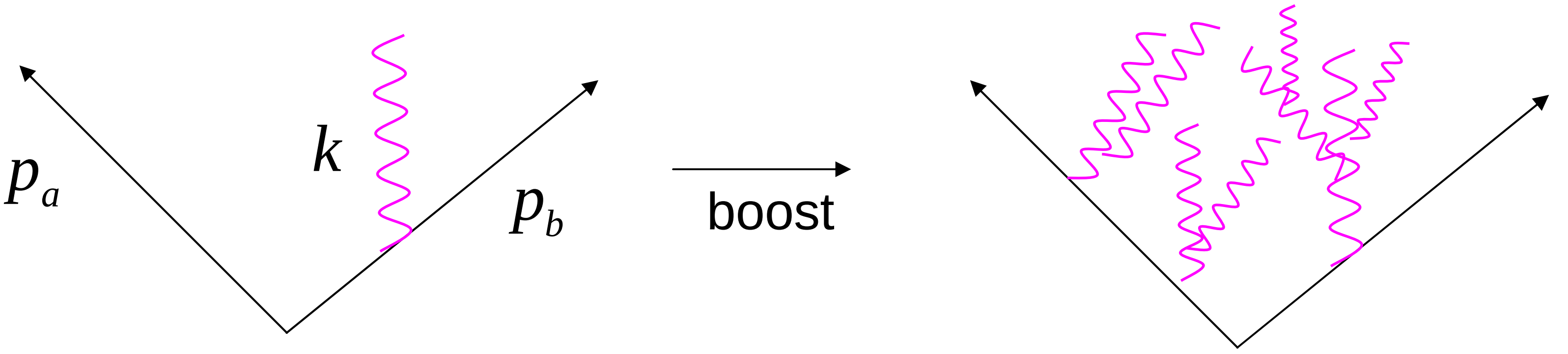
Sudakov logs. (emission from primary partons)
Kidonakis, Oderda, Sterman (1997)
Non-global logs. (emission from secondary gluons)
Dasgupta, Salam (2001)

Marchesini-Mueller equation

Marchesini, Mueller (2003)

Differential probability for the soft gluon emission

$$dP = \bar{\alpha}_s \omega d\omega \frac{d\Omega_k}{4\pi} \frac{p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} \approx \bar{\alpha}_s \frac{d\omega}{\omega} \frac{d\Omega_k}{4\pi} \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})}$$



Evolution of the interjet gluon number. Non-global logs included.

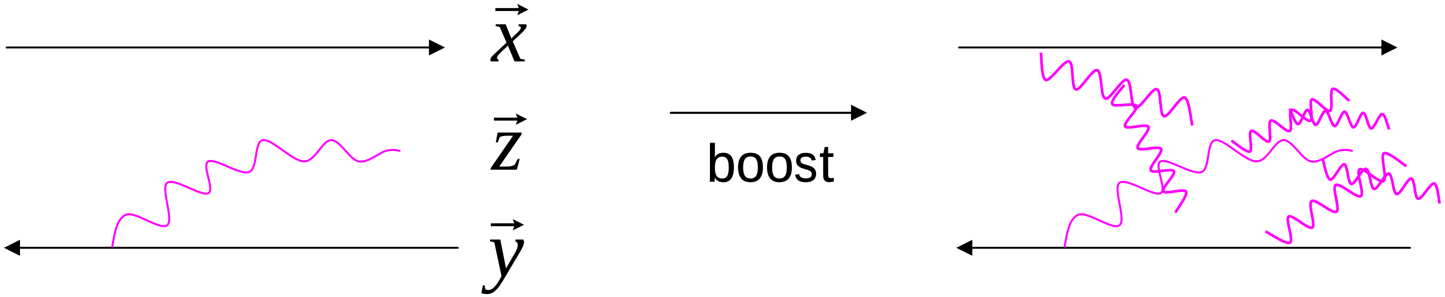
$$\partial_Y n(\theta_{ab},\theta_{cd},Y) = \bar{\alpha}_s \int \frac{d^2\Omega_k}{4\pi} \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})} \times \left(n(\theta_{ak},\theta_{cd},Y) + n(\theta_{bk},\theta_{cd},Y) - n(\theta_{ab},\theta_{cd},Y)\right).$$

$Y = \ln 1/x$

BFKL equation

Differential probability for the dipole splitting

$$dP = \overline{\alpha}_s \frac{d\omega}{\omega} d^2\vec{z} \frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2(\vec{z}-\vec{y})^2}$$

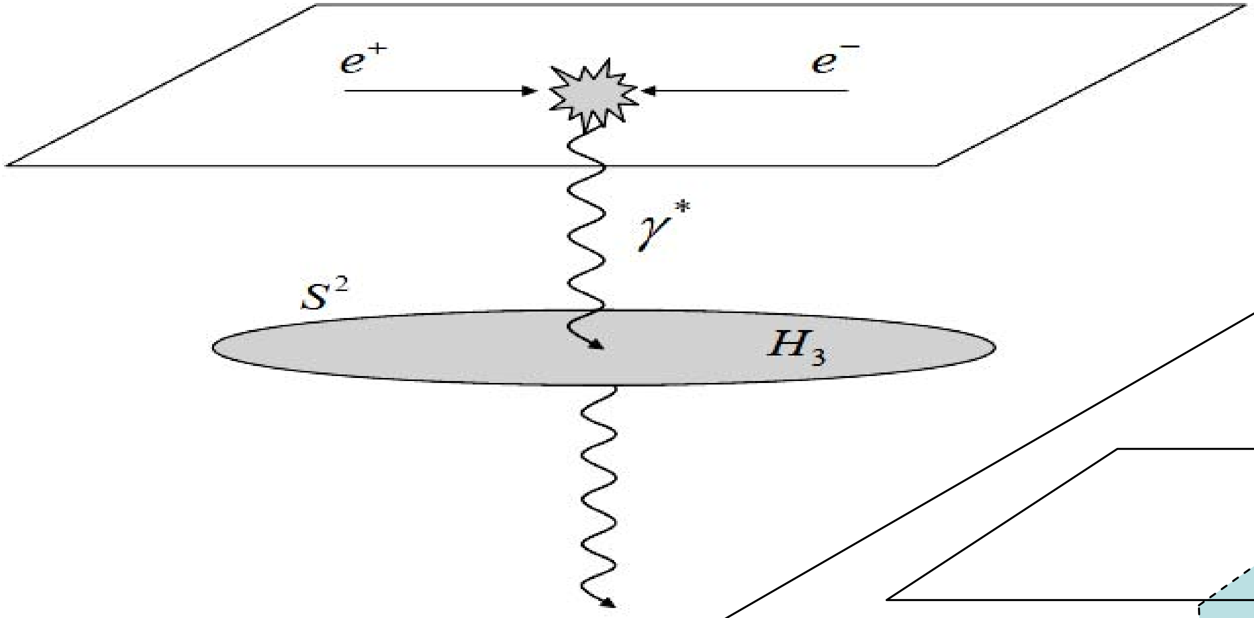


Dipole version of the BFKL equation Mueller (1995)

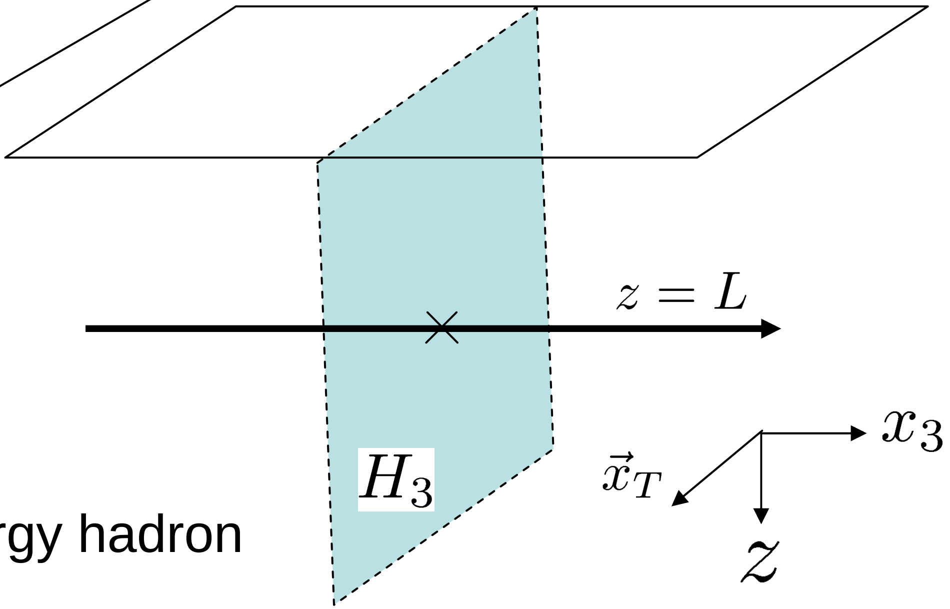
$$\begin{aligned}\partial_Y n(x_{ab}, x_{cd}, Y) &= \bar{\alpha}_s \int \frac{d^2\vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2} \\ &\times \left(n(x_{ak}, x_{cd}, Y) + n(x_{bk}, x_{cd}, Y) - n(x_{ab}, x_{cd}, Y)\right)\end{aligned}$$

A hint from AdS/CFT

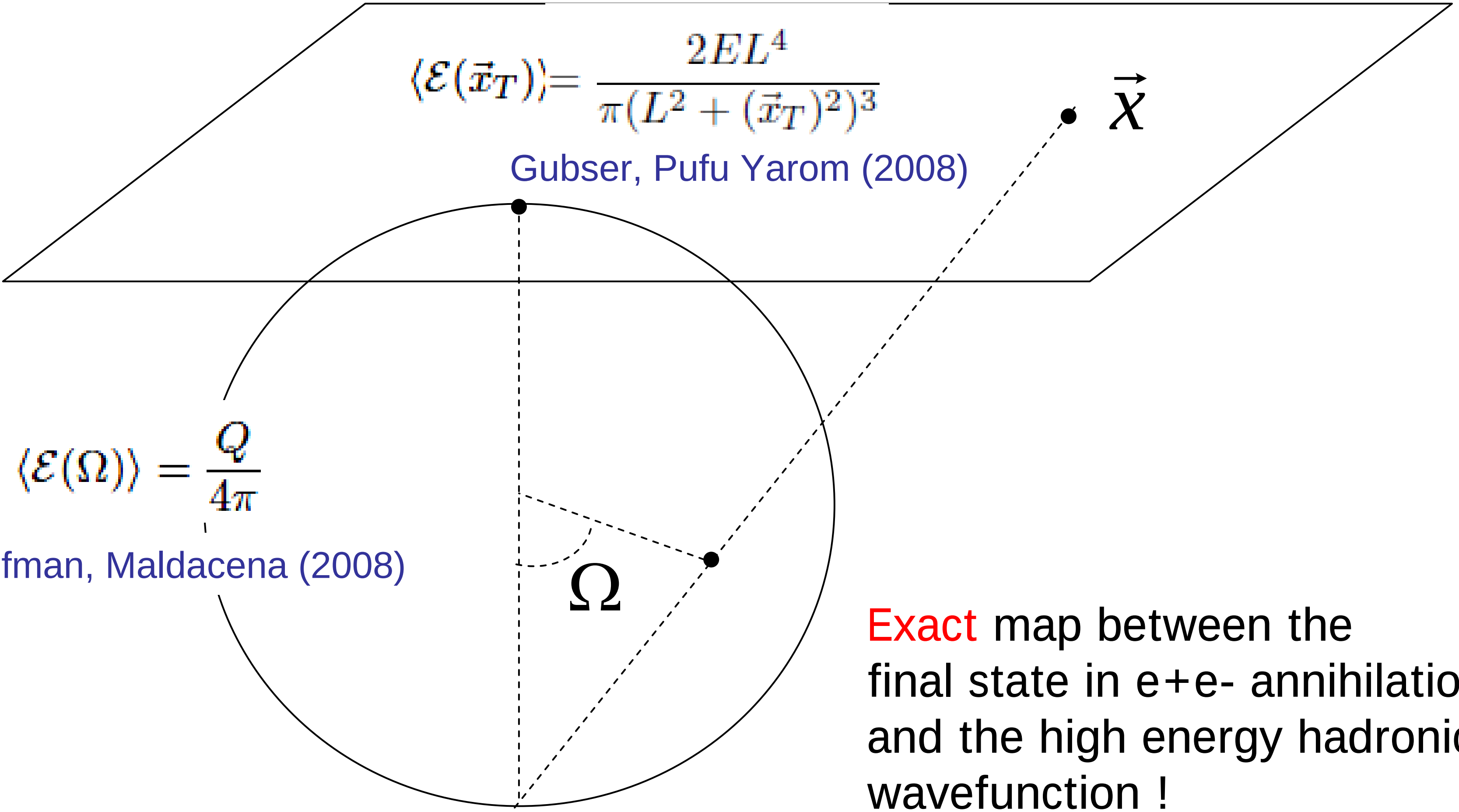
Final state of e+e- annihilation



Transverse profile a high energy hadron



The stereographic map



$$\langle \mathcal{E}(\vec{x}_T) \rangle = \frac{2EL^4}{\pi(L^2 + (\vec{x}_T)^2)^3}$$

Gubser, Pufu Yarom (2008)

$$\langle \mathcal{E}(\Omega) \rangle = \frac{Q}{4\pi}$$

Hofman, Maldacena (2008)

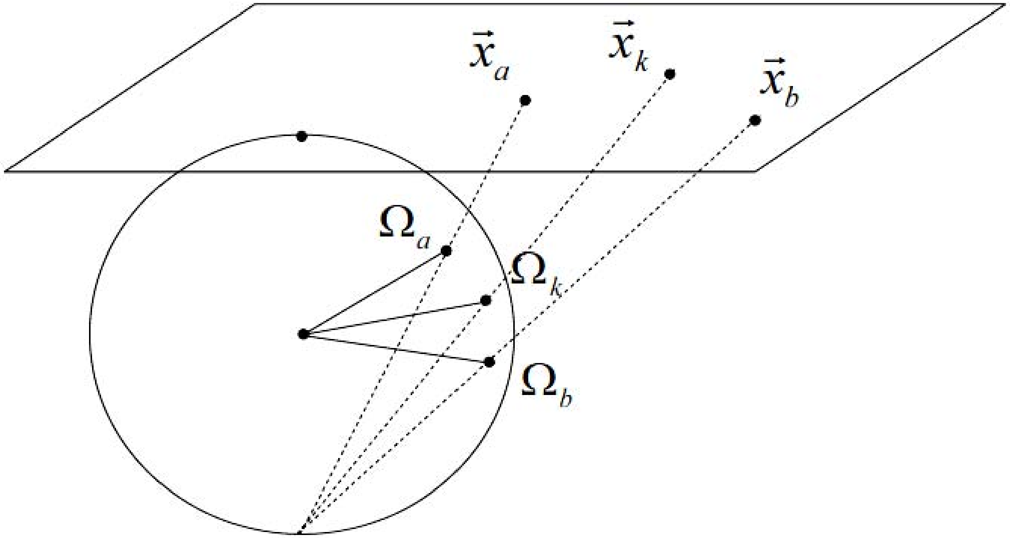
Exact map between the final state in e+e- annihilation and the high energy hadronic wavefunction !

YH (2008)

Exact map at weak coupling

The same stereographic map transforms BFKL into Marchesini-Mueller

$$\frac{d^2\Omega_k}{4\pi}\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})}=\frac{d^2\vec{x}_k}{2\pi}\frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$



Make the most of conformal symmetry SL(2,C) of the BFKL kernel.
Exact solution to the Marchesini—Mueller equation YH (2008)
and much more ! Avsar, YH, Matsuo (2009)

NLL timelike dipole evolution in N=4 SYM

Avsar, YH, Matsuo (2009)

Apply the stereographic projection
to the result by Balitsky & Chirilli (2008).

$$\partial_Y n_Y(\Omega_{ab}) = \bar{\alpha}_s \left(1 - \bar{\alpha}_s \frac{\pi^2}{12}\right) \int d^2\Omega_c\, K_{ab}(\Omega_c) \left[n_Y(\Omega_{ac}) + n_Y(\Omega_{cb}) - n_Y(\Omega_{ab})\right]$$

$$+\bar{\alpha}_s^2 \int d^2\Omega_c d^2\Omega_d K'_{ab}(\Omega_c, \Omega_d) n_Y(\Omega_{cd})\,,$$

$$\begin{aligned} K'_{ab}(\Omega_c, \Omega_d) = \frac{1}{8\pi^2} \Bigg\{ & \frac{(1-\cos\theta_{ab})}{(1-\cos\theta_{ac})(1-\cos\theta_{cd})(1-\cos\theta_{db})} \\ & \times \left[\left(1 + \frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ac})(1-\cos\theta_{bd}) - (1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right) \right. \\ & \left. \times \ln \frac{(1-\cos\theta_{ac})(1-\cos\theta_{bd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} + 2 \ln \frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right] \\ & + 12\pi^2 \zeta(3) \delta^{(2)}(\Omega_{ac}) \delta^{(2)}(\Omega_{bd}) \Bigg\}\,. \end{aligned}$$

Conclusion

- AdS/CFT offers a new approach to nonperturbative aspects of high energy scattering.
- Some results completely different from QCD, some are useful.
- Feedback to weakly coupled, approximately conformal QCD. Certain features are universal !