超対称QCDと
閉じ込めの位相的側面

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講演内容

- クォークの閉じ込めとトポロジカルソリトン
- 超対称模型による理解：Abelian superconductor
- 超対称模型による理解：non-Abelian superconductor
- monopole dynamics from flux tube
- non-Abelian superconductor in high density QCD
- conclusion

クォークの閉じ込めと
トポロジカルソリトン

閉じ込め問題

QCD: strong interaction

microscopic degrees of freedom

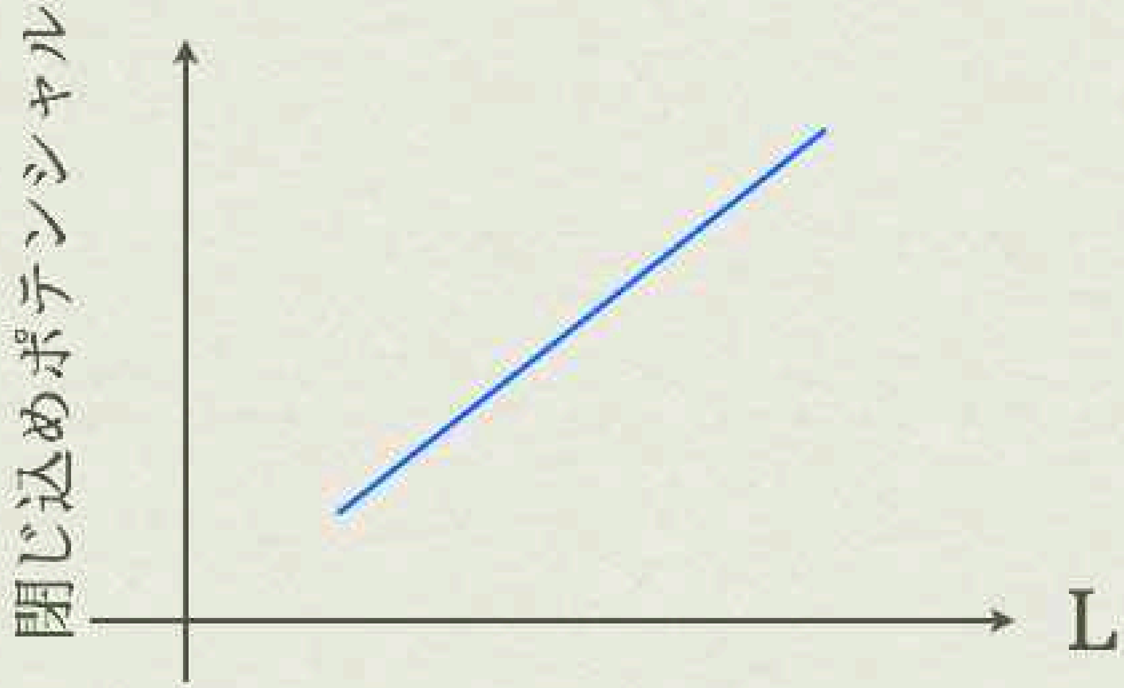
quarks and gluons

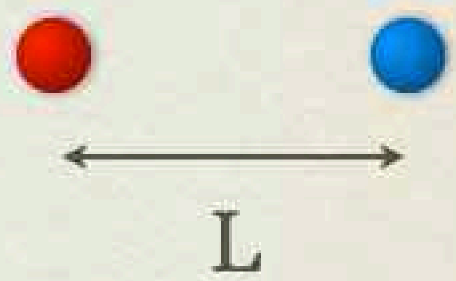
But they have been never directly observed so far.

At low energy, we observe only hadrons.

低エネルギーではカラー自由度が見えない

Why? ⟶ linear confinement in QCD

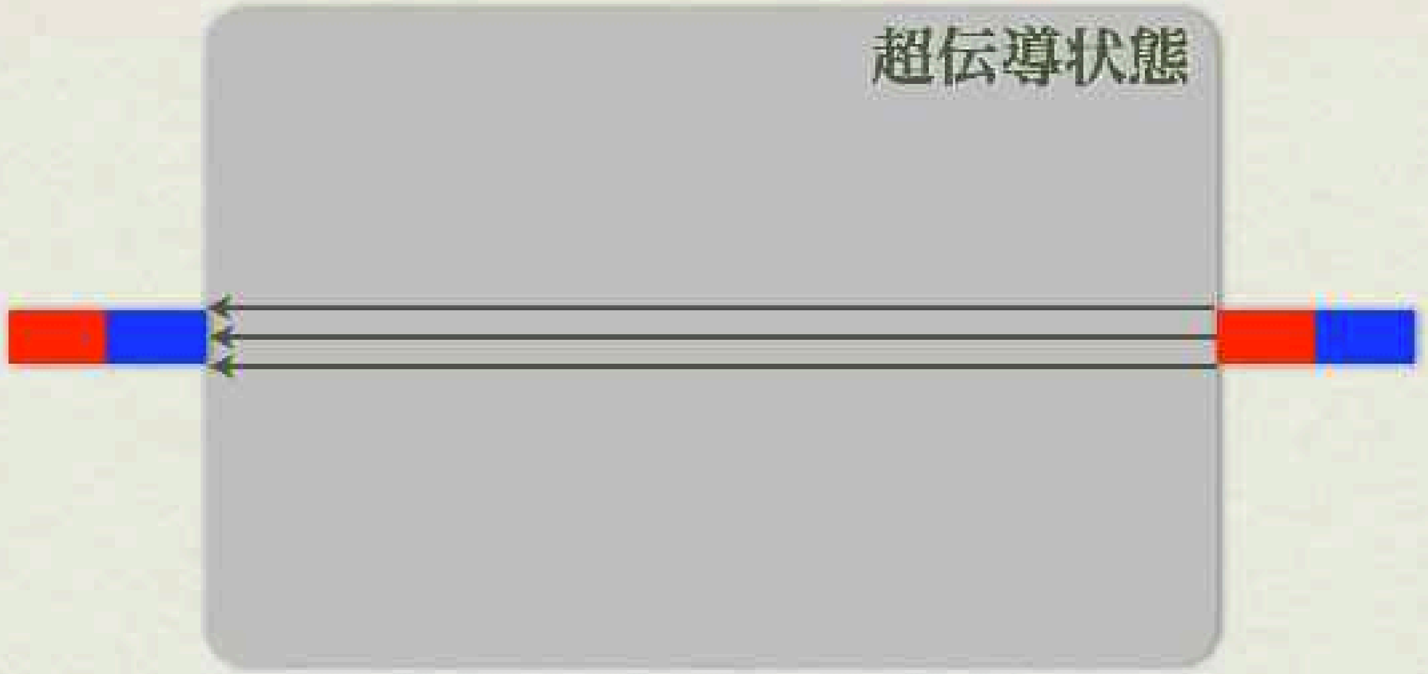




The strong force increase (not decrease) as quarks separate!!

Do we know such a fundamental force in Nature?

金属の超伝導



Meissner effect

Magnetic flux tube (string) forms in superconductor

potential = tension × length

金属の超伝導 aspects of symmetry

BCS theory

$\boxed{\text{electrons form a Cooper pair and condense}}$

★ Electrically charged objects condense

★ U(1) gauge symmetry is spontaneously broken

(Higgs mechanism: photon becomes massive)

★ Magnetic flux is squeezed in supercond.

★ Magnetic monopoles are confined

Nambu-'t Hooft-Mandelstam idea

Electromagnetic duality

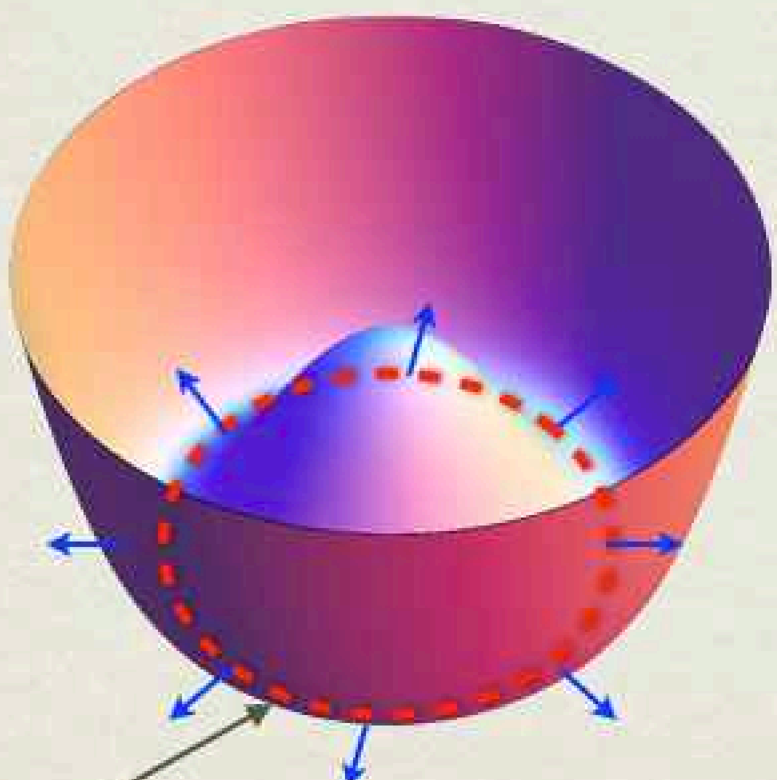
	Meissner	dual Meissner
gauge symm.	$U(1)_{electric}$	$U(1)_{magnetic}$
charge	e	g (~1/e)
condensation	electron	magnetic monopole
flux tube	magnetic flux	electric flux

Nielsen-Olesen vortex

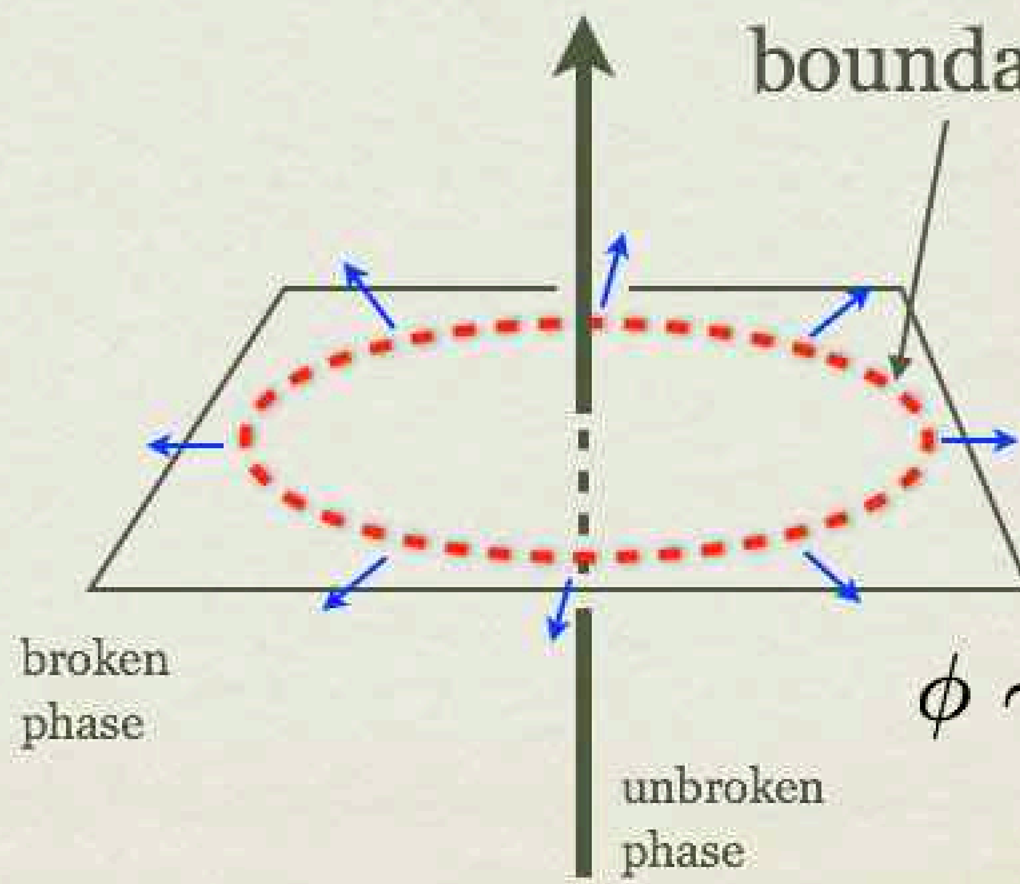
Abelian-Higgs theory $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu \phi|^2 - V(\phi)$

(dual) U(1) symmetry is spontaneously broken

Topologically non-trivial breaking $\longrightarrow$ topological flux tube



boundary = S^1



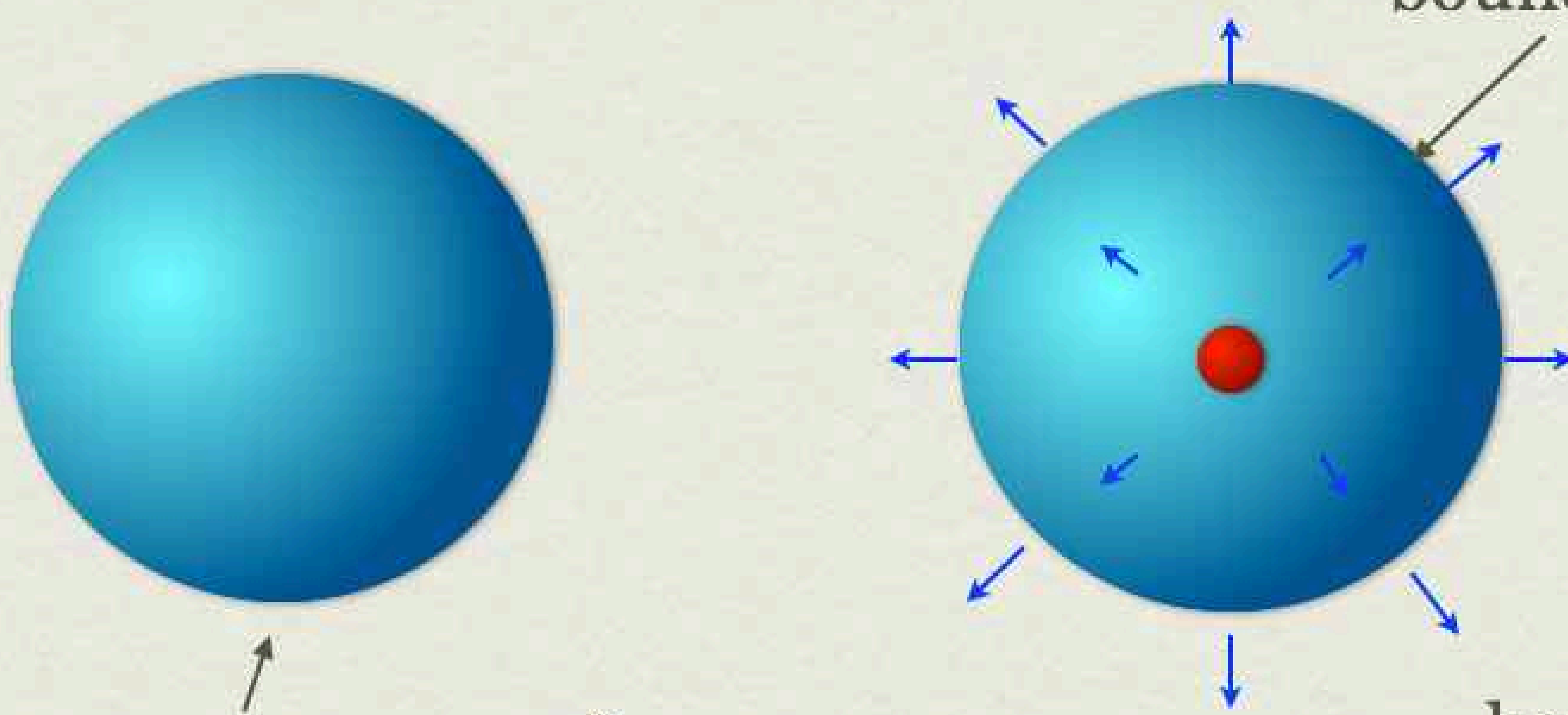
$\phi \sim v e^{i\theta}$

vac structure = S^1 $\pi_1(U(1)) = \pi_1(S^1) = \mathbb{Z}$

‘t Hooft-Polyakov magnetic monopole

$SO(3) \simeq SU(2)/Z_2$ Yang-Mills - Higgs theory

$$V(\phi_a) = \lambda \left(\phi_a^2 - v^2\right)^2 \qquad (a = 1, 2, 3)$$



$$\pi_2(SU(2)/U(1)) = \pi_2(S^2) = \mathbb{Z}$$

To clarify the confinement in QCD...

It is important to understand topological objects in QCD.

But it is very difficult since QCD is asymptotically free and perturbative analysis cannot be applied in low energy.

Many approaches

- lattice simulation
- large N
- SUSY
- gravity dual

large N, SUSY, gravity dual: QCD-like theories

超対称性による理解

ABELIAN SUPERCONDUCTOR

SUSY

Milestone Seiberg-Witten (`94)

The first theory in which one can analytically show the dual Meissner effect occurs in low energy.

SUSY: holomorphy gives strong constraints on theories

superpotential: $W = W(\Phi)$

prepotential: $\mathcal{F} = \mathcal{F}(A)$

(Φ, A complex filed)

Exact treatments are possible even in strongly coupled theories.

N=2 pure SYM (Seiberg-Witten)

	A_μ	....	SU(2) gauge field
vector multiplet	$\lambda \quad \psi$	....	two Weyl fermions
	ϕ	....	complex scalar fields

Classical potential

$$V = \frac{1}{g^2}\mathrm{Tr}\left[\phi, \phi^\dagger\right]^2$$

$$\phi = \frac{a}{2}\sigma_3 \quad SU(2) \to U(1)$$

a: complex parameter (local coordinate of the moduli space)

U(1) effective theory (by N=1 language)

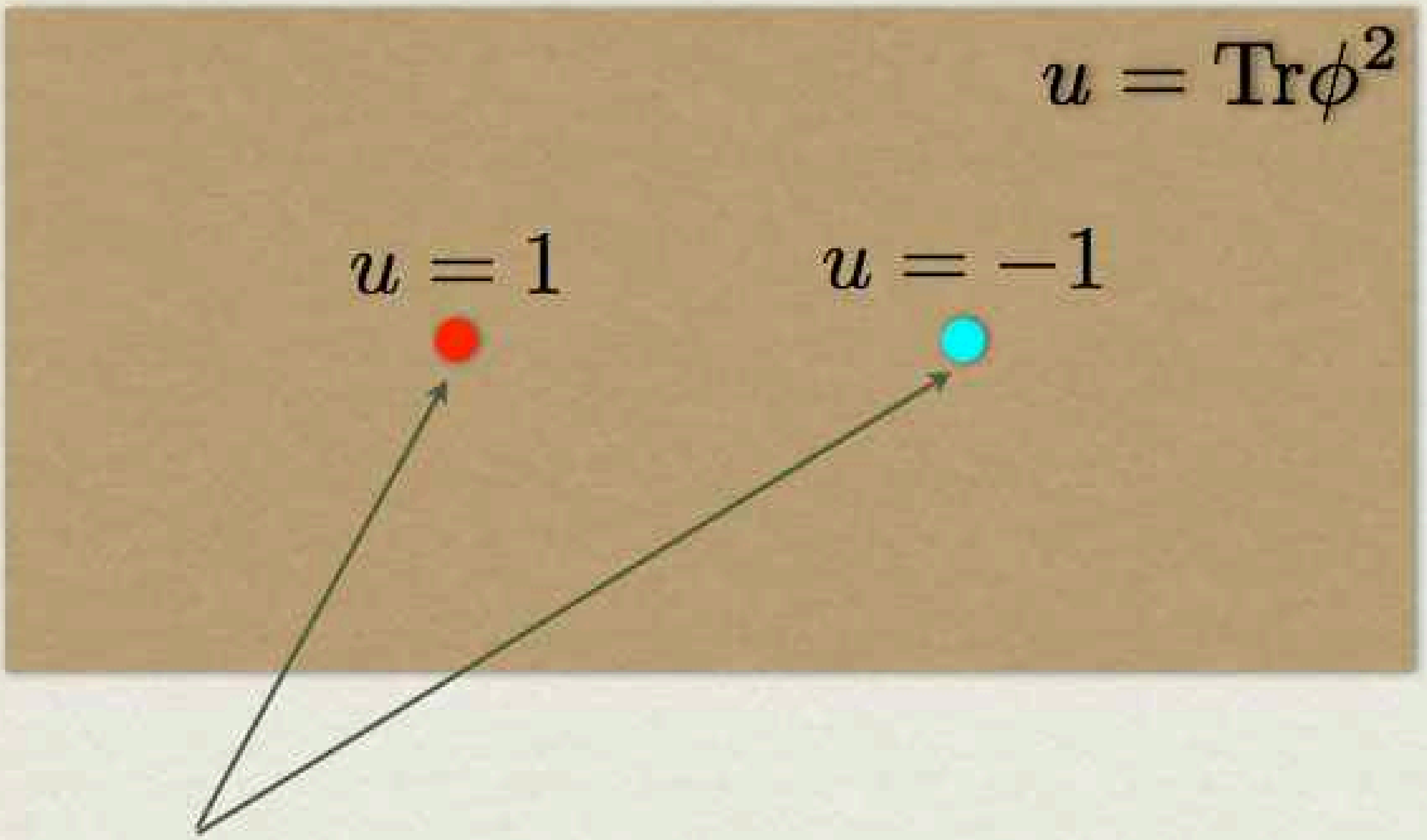
$$\mathcal{L} = \frac{1}{4\pi}\mathrm{Im}\left[\int d^4\theta \frac{\partial \mathcal{F}}{\partial a}\bar{a} + \int d^2\theta \frac{1}{2}\frac{\partial^2 \mathcal{F}}{\partial a^2} W_\alpha W^\alpha\right]$$

metric of the moduli space $ds^2 = \mathrm{Im}\frac{\partial^2 \mathcal{F}}{\partial a^2} da d\bar{a}$

$$\mathcal{F}_{\mathrm{classical}}(a) = \frac{1}{2}\left(i\frac{4\pi}{g^2} + \frac{\theta}{2\pi}\right)a^2$$

Seiberg-Witten's work: Determine exact form of $\mathcal{F}(a)$

quantum moduli space of N=2 SU(2) pureSYM



u=∞:electric description
u<∞:magnetic description

two singularities: massless monopole and dyon!!

	η	…………	Weyl fermion
hypermultiplet	M $\tilde{M}$	…………	two complex scalar
(in dual description)	$\tilde{\eta}$	…………	Weyl fermion

SUSY QCD

Softly break N=2 SUSY to N=1 SUSY by adding mass

A_μ ⟵ N=1 vector multiplet

N=2 gauge multiplet λ ψ

ϕ ⟵ N=1 chiral multiplet (massive)

This can be done by adding the superpotential:

$$W(\Phi) = m\mathrm{Tr}\Phi^2$$

N=2 SUSY determine the effective potential near u=1:

$$W_D = \sqrt{2} a_D M \tilde{M} + m U(a_D)$$

Vaccum condition: $dW_D = 0$

monopole condensation

$$\langle M \rangle = \left\langle \tilde{M} \right\rangle \neq 0$$

electric charge confinement!!

PURE SU(3)

Adjoint field in N=2 vector multiplet

$$\Phi = a_3 T_3 + a_8 T_8$$

In general, the symmetry breaking is

$$SU(3) \longrightarrow U(1) \times U(1)$$

Low energy magnetic theory is a dual U(1) × U(1) gauge theory.

$$\begin{cases} \text{two different Abelian monopoles appear} \\ \text{two different confining string appears} \end{cases}$$

$\longrightarrow$ too rich meson spectrum

[Douglas-Shenker (`95)]

超対称性による理解

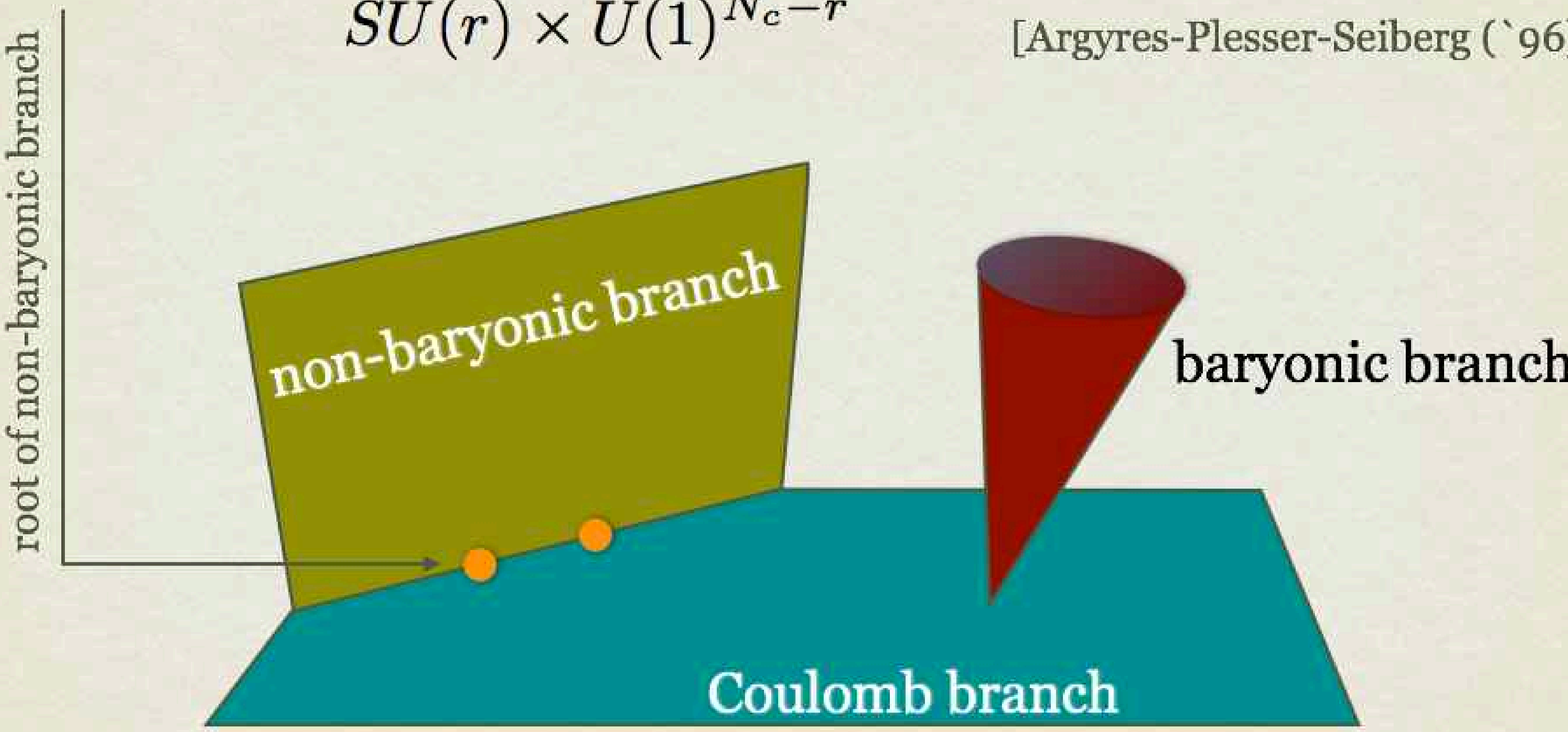
NON-ABELIAN SUPERCONDUCTOR

SU(3)+ FLAVORS

r-vacuum: non-Abelian gauge symmetry survives in dual theory

$$SU(r) \times U(1)^{N_c - r}$$

[Argyres-Plesser-Seiberg (`96)]



quantum moduli space of N=2 SU(N) QCD with Nf flavors

Number of flavors & non-Abelian dual Meissner effect

r-vacuum: $r \leq \left[\frac{N_f}{2}\right]$

asymptotic free in microscopic theory: $N_f < 2N_c$

IR free in macroscopic theory: $r \leq \left[\frac{N_f}{2}\right]$

We are interested in $N_c=3$ and $r=2$, so $N_f=4,5$.

r=2 vacuum: $SU(2) \times U(1)$ dual gauge theory

w/ N_f dual quark multiplet in fundamental rep.

identified with non-Abelian monopole

[Goddard-Nuyts-Olive (`77)]
[Bolognesi-Konishi (`02)]

$SU(3) \rightarrow SU(2) \times U(1) \quad \pi_2(G/H) = \pi_1(H) = \mathbb{Z}$

SEMI-CLASSICAL ANALYSIS

[Auzzi-Bolognesi-Evslin-Konishi-Yung (`03)]

non-Abelian monopole condensation: non-Abelian Meissner effect

non-Abelian monopole
[Goddard-Nuyts-Olive (`77)]

non-Abelian flux tube

What is this?

It is not Abelian (Nielsen-Olesen) flux.

It should posses color and flavor degrees of freedom!

[Auzzi-Bolognesi-Evslin-Konishi-Yung (`03)]

[Hanany-Tong (`03)]

softly broken N=2 SU(3) SYM + Nf=4,5 flavors:

[Auzzi-Bolognesi-Evslin-Konishi-Yung (`03)]

superpotential

$$W = \sqrt{2}\mathrm{Tr}_c \left[\underbrace{Q\tilde{Q}\Phi + QM\tilde{Q}}_{\text{N=2 structure}} + \underbrace{\mu\Phi^2}_{\text{soft breaking}} \right]$$

$Q, \tilde{Q}$: hypermultiplet	equal masses
A_μ, Φ : vector multiplet	$M = m\mathbf{1}_{N_f}$

$$Q = \begin{pmatrix} Q_{11} & \cdots & Q_{1N_f} \\ \vdots & & \vdots \\ Q_{N_f 1} & \cdots & Q_{N_c N_f} \end{pmatrix}$$ Nc × Nf matrix

semi-classical analysis is justified $\Lambda \ll \mu \ll m$

$\boxed{\text{scale at } m}$

(non-Abelian magnetic monopole)

$$SU(3) \to SU(2) \times U(1)$$

$$\langle \Phi \rangle = m T_8$$

$\boxed{\text{scale at } \mu \ll m}$

heavy fields with mass m are integrated out.

$$\text{W-bosons \& } Q = \begin{pmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \end{pmatrix}$$

light fields

massive

Low energy effective theory: U(1)×SU(2) SYM w/ Nf=4 flavors

$$W_{\mathrm{LET}}^{SU(2)} = \sqrt{2}\mathrm{Tr}_c\left[q\tilde{q}\phi_a T_a\right] \quad W_{\mathrm{LET}}^{U(1)} = \sqrt{2}\mathrm{Tr}_c\left[q\tilde{q}\phi_0 - \mu m \mathbf{1}_2\right]$$

squark fields q develop non-zero VEV

$$q = \sqrt{\mu m} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

U(1)×SU(2) gauge symmetry is completely broken!!

$$SU(3) \underset{m}{\rightarrow} U(1) \times SU(2) \underset{\sqrt{\mu m}}{\rightarrow} \mathbf{1}$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ \text{monopole} & & \text{vortex} \\ \downarrow & & \downarrow \end{array}$$

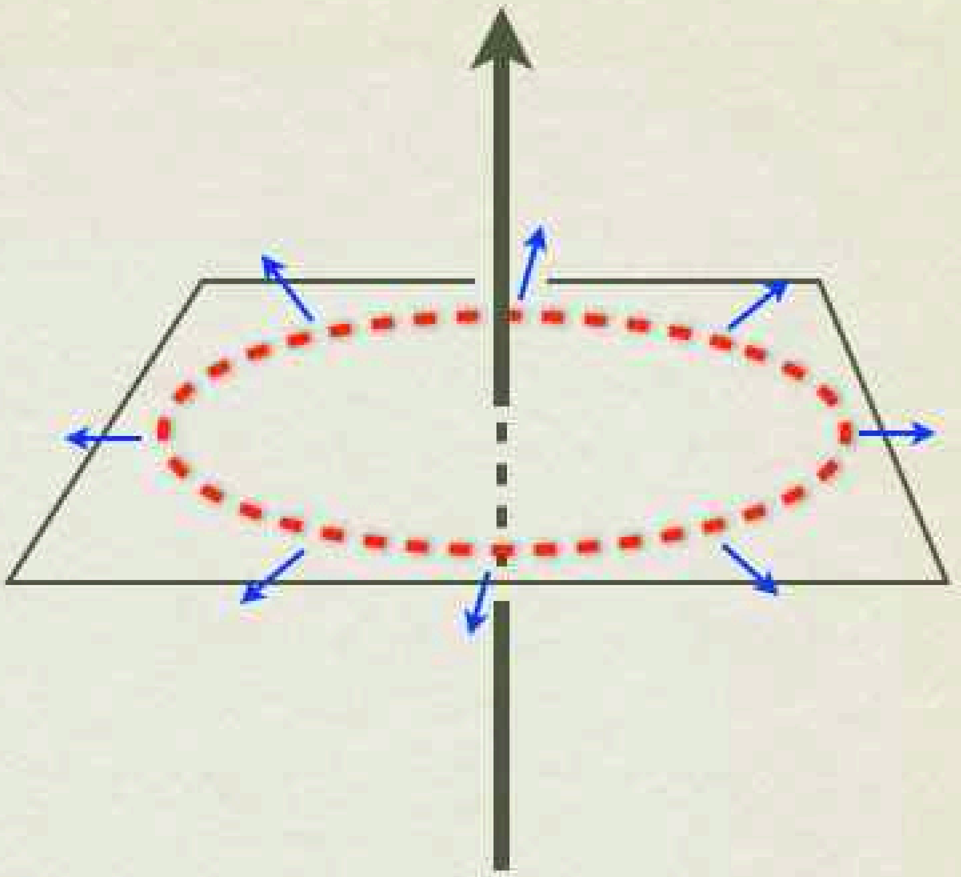
$$\pi_2(G/H) = \pi_1(H) = \mathbb{Z}$$

What is non-Abelian flux tube?

vacuum is color-flavor locking

$$SU(2)_{c+f} \times SU(2)_f$$

$$q = \sqrt{\mu m} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$



Vortex solution:

$$q = \sqrt{\mu m} \begin{pmatrix} f(r)e^{i\theta} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$\Big\downarrow$ center of vortex

$$q = \sqrt{\mu m} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad U(1)_{c+f} \times SU(2)_f$$

NG modes (orientational zero modes) $\quad \mathbb{C}P^1 \simeq \dfrac{SU(2)_{c+f}}{U(1)_{c+f}}$

[Auzzi-Bolognesi-Evslin-Konishi-Yung (`03)]

RELATED WORKS

Moduli space of BPS solitons

integrability system

Instanton / Monopoles by ADHM / Nahm construction

non-integrability system

Vortex / Domain walls by Moduli Matrix Formalism

[Isozumi-Nitta-Ohashi-Sakai(`04)] [Eto-Isozumi-Nitta-Ohashi-Sakai(`06)] [many]

Cosmic string

[Eto-Hashimoto-Marmorini-Nitta-Ohashi-Vinci(`06)]

Relation to non-Abelian monopole

[Eto-Konishi-Marmorini-Nitta-Ohashi-Vinci-Yokoi(`06)]

Other gauge group SO/USp etc

[Eto-Fujimori-Gudnason-Konishi-Nitta-Ohashi-Vinci(`09)]

MONOPOLE DYNAMICS FROM FLUX TUBE

MORE ON MONOPOLE CONFINEMENT

Let us next consider the monopole in low energy theory:

We add a new scale δm in such a way

$$U(1) \times SU(2) \xrightarrow{\delta m} U(1) \times U(1) \xrightarrow{\sqrt{\mu m}} \mathbf{1}$$

Abelian monopole   Abelian votex

Origin of δm is traced into the microscopic SU(3) theory

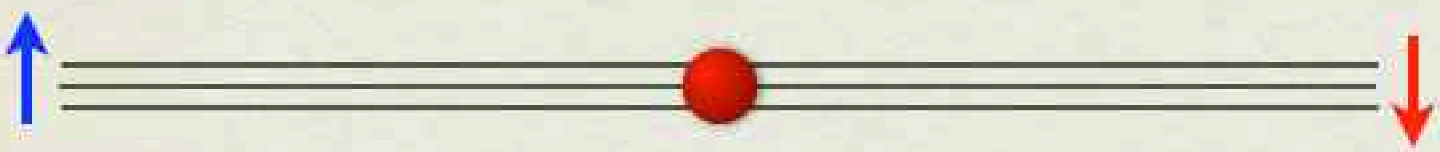
$$M = m\mathbf{1}_{N_f} \longrightarrow M = \mathrm{diag}(m + \delta m, m - \delta m, m, m)$$

Abelian monopole confinement is also under control semi-classically and quantum mechanically.

[Tong (`03)] [Hanany-Tong (`03)] [Shifman-Yung (`04)] [Isozumi et al(`04)]

Confined monopole

not static and difficult to treat



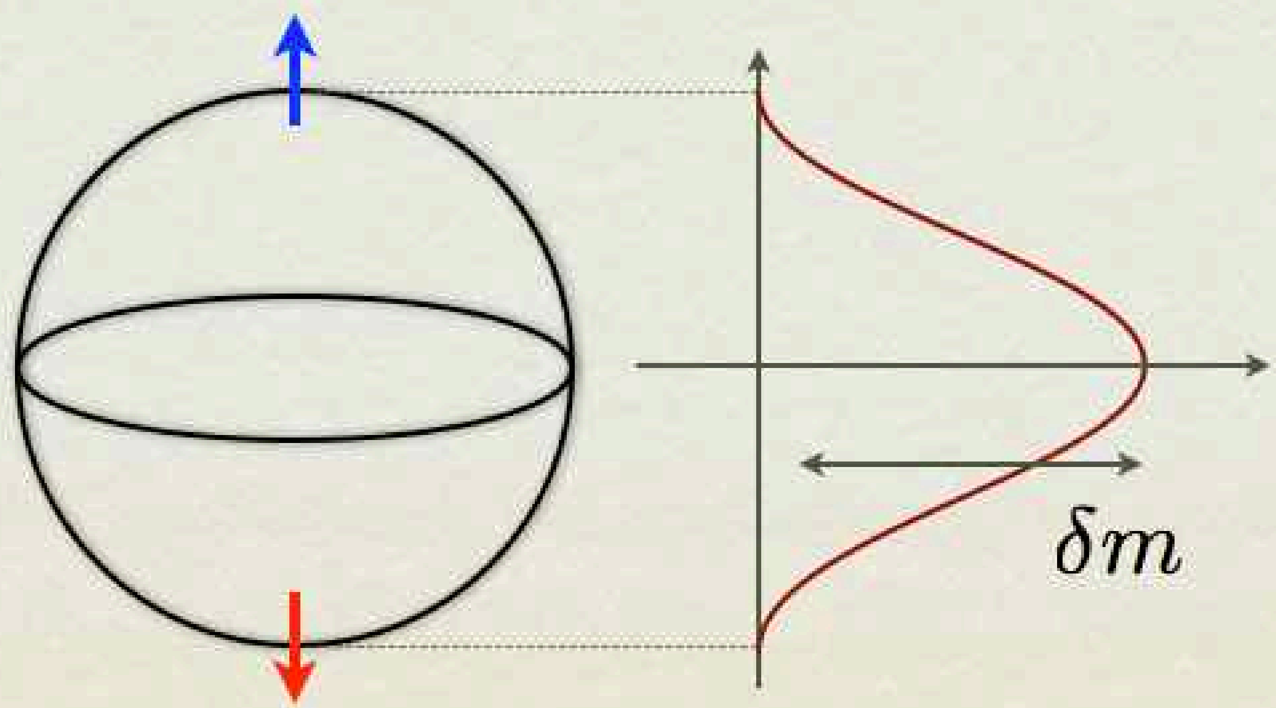
static confined monopole

(easy) [Tong (`03)]

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \in u(2) \qquad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \in u(2)$$

1/4 BPS state!

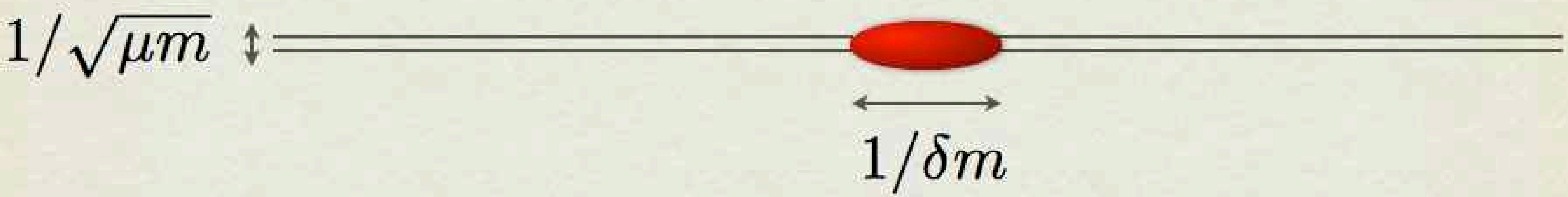
$$\mathbb{C}P^1 = S^2$$



Monopole as kink on vortex [Tong (`03)]

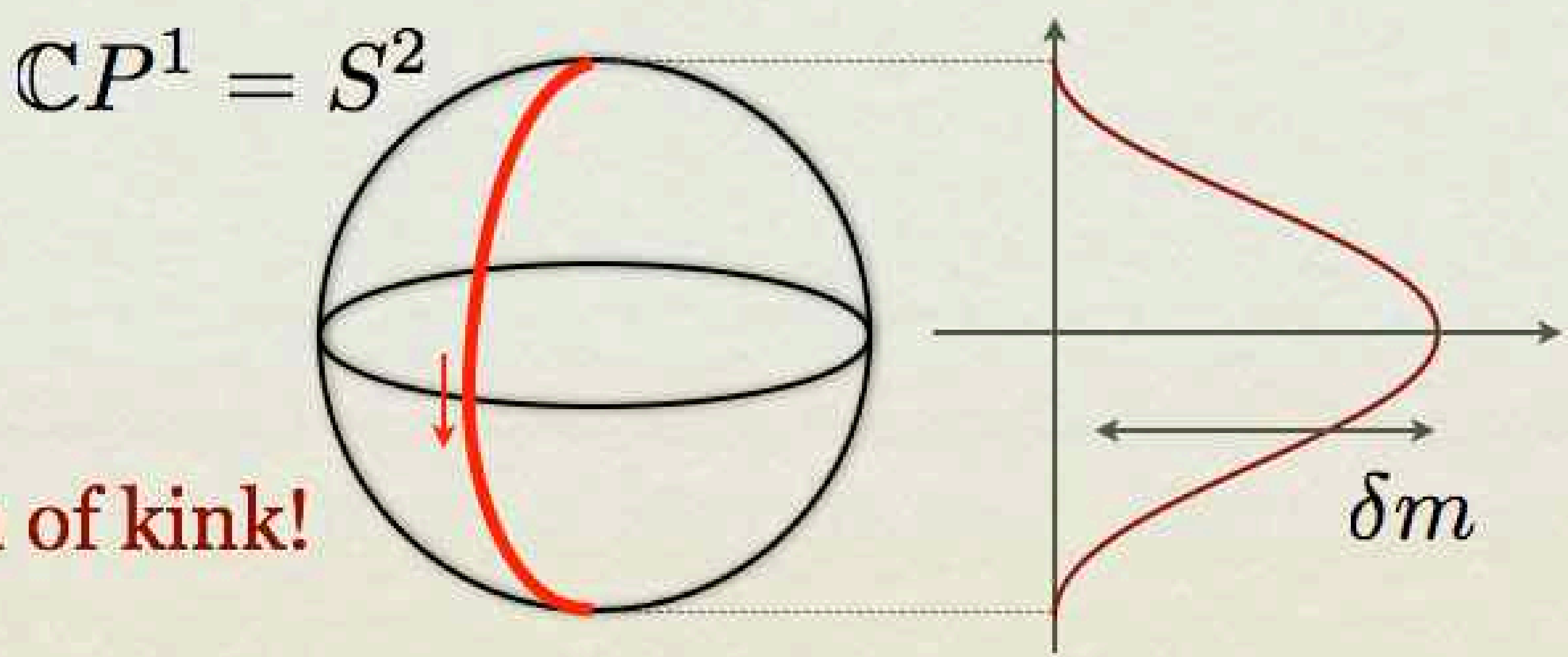
Let us consider the case $\sqrt{\mu m} \gg \delta m$

(Firstly, vortex forms, then confined monopole forms)



1+1 dim. effective theory on vortex world-volume

= 1+1 dim. massive CP(1) NLSM



exact solution of kink!

Dynamics of monopoles [Arai-Blashcke-Eto-Sakai (in preparation)]

So far, only static solutions are studied.

How about dynamics of monopoles!?

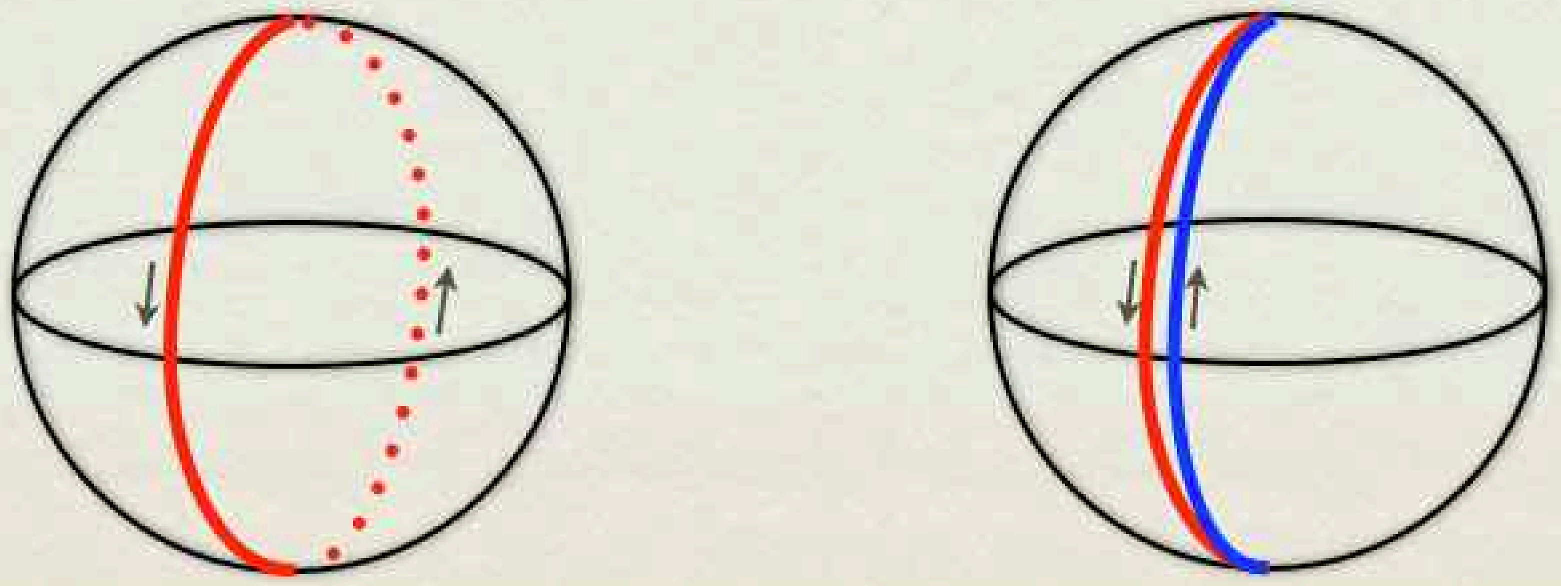
Does vortex give us some advantage?

YES!

A great circle of CP(1) = sine-Gordon model (integrable system)

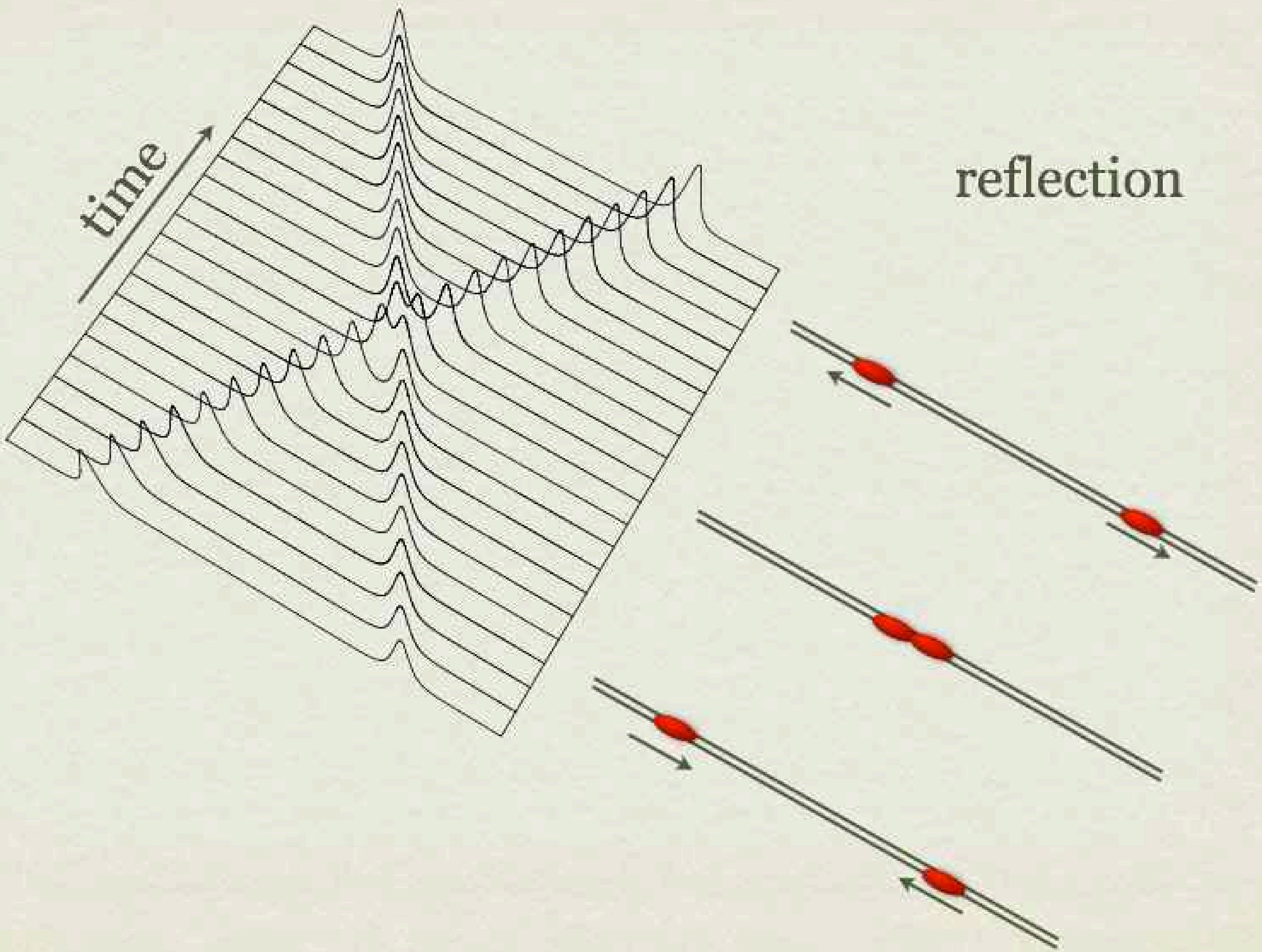
Exact solutions for kink-kink / kink-antikink / breather

our interpretation: kink = monopole!

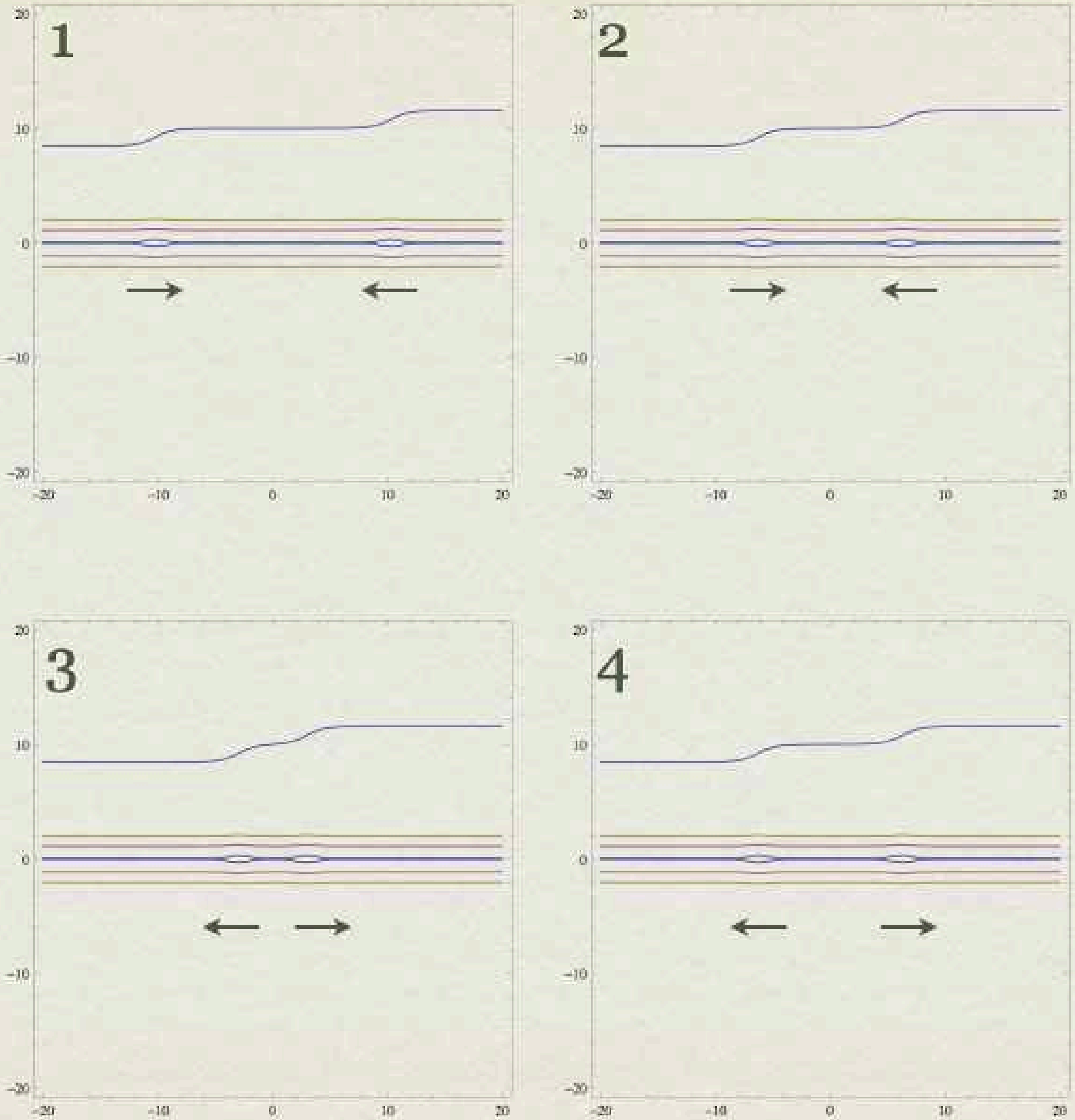


kink - kink   kink - antikink

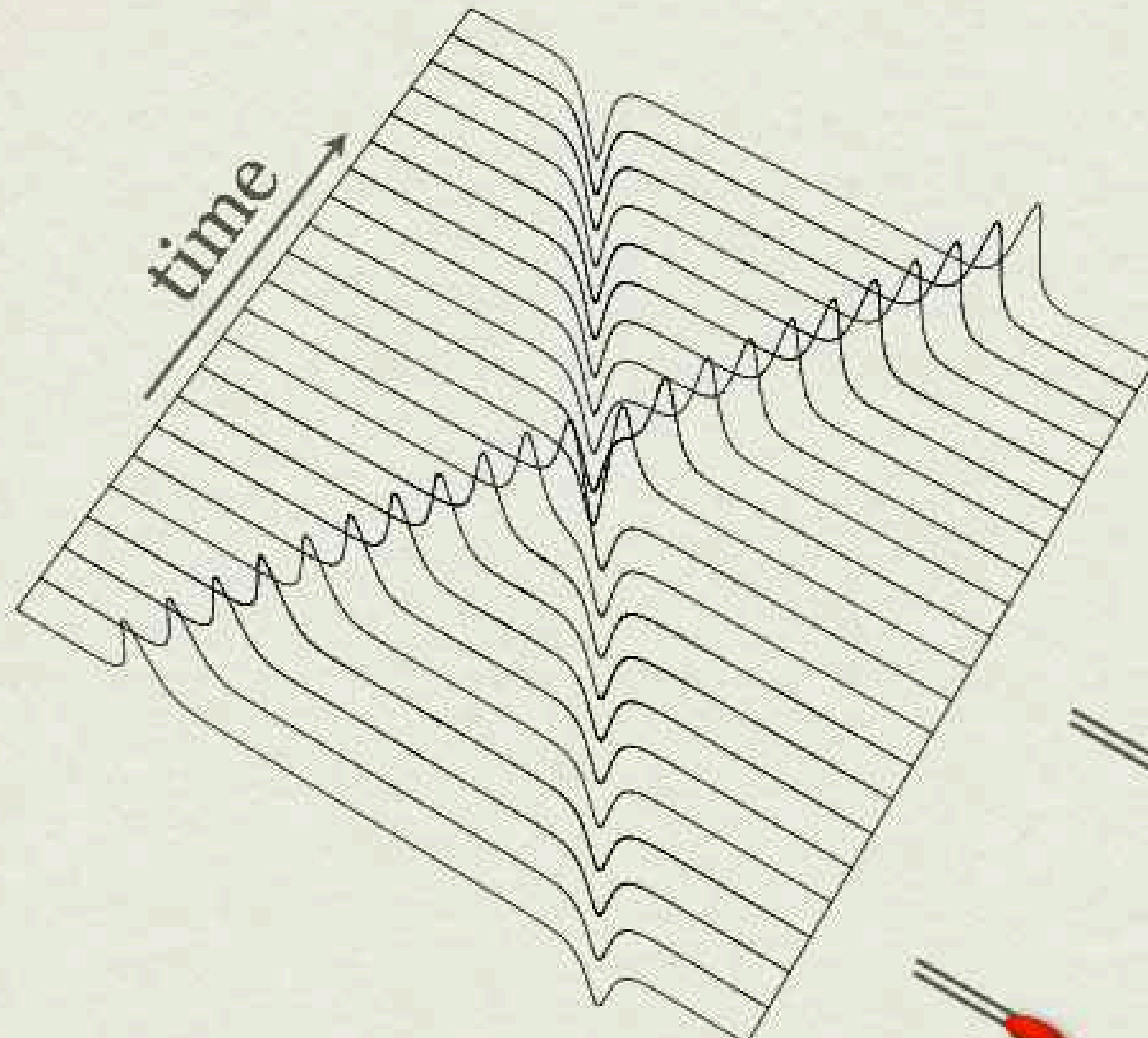
kink-kink



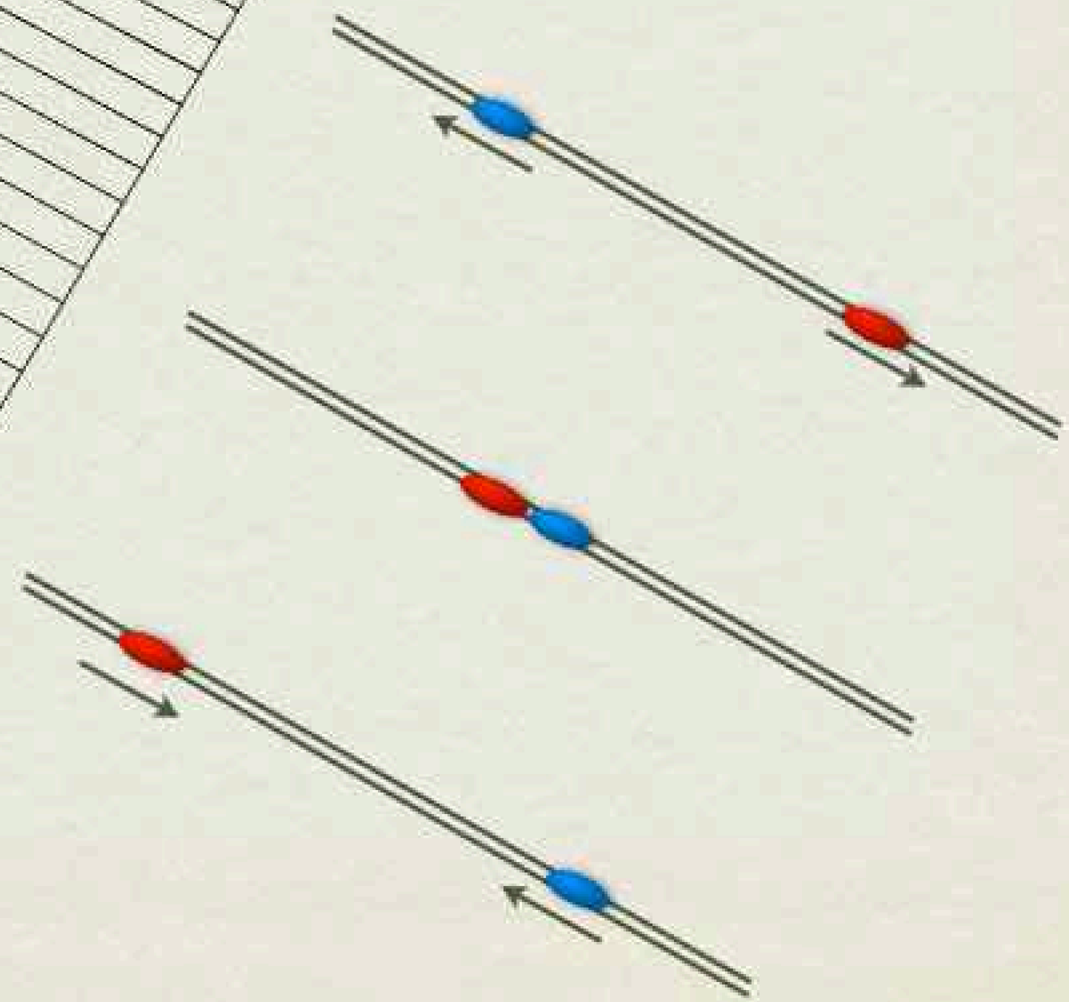
reflection

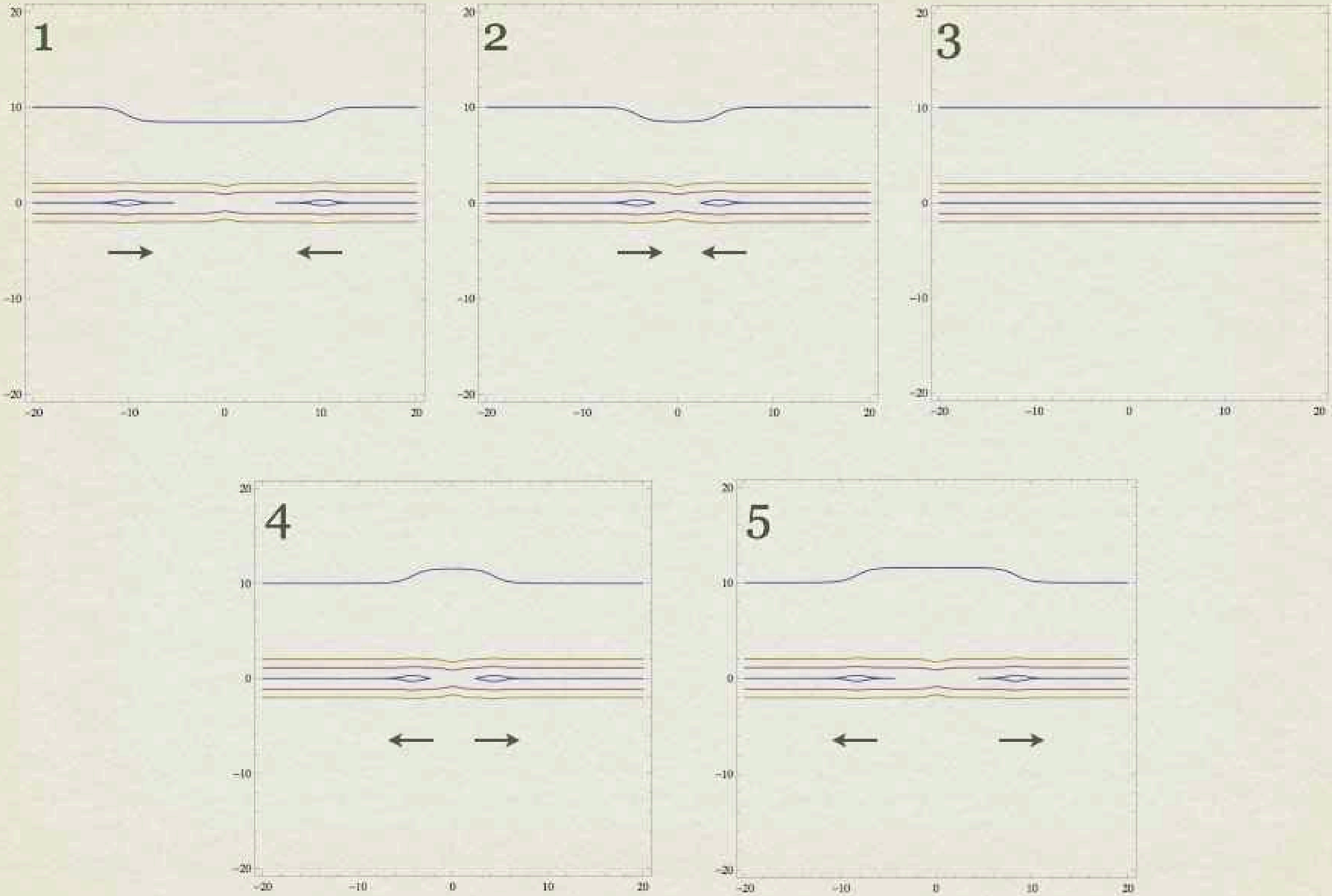

1
2
3
4
20
10
0
−10
−20
−20
−10
0
10
20

kink-antikink

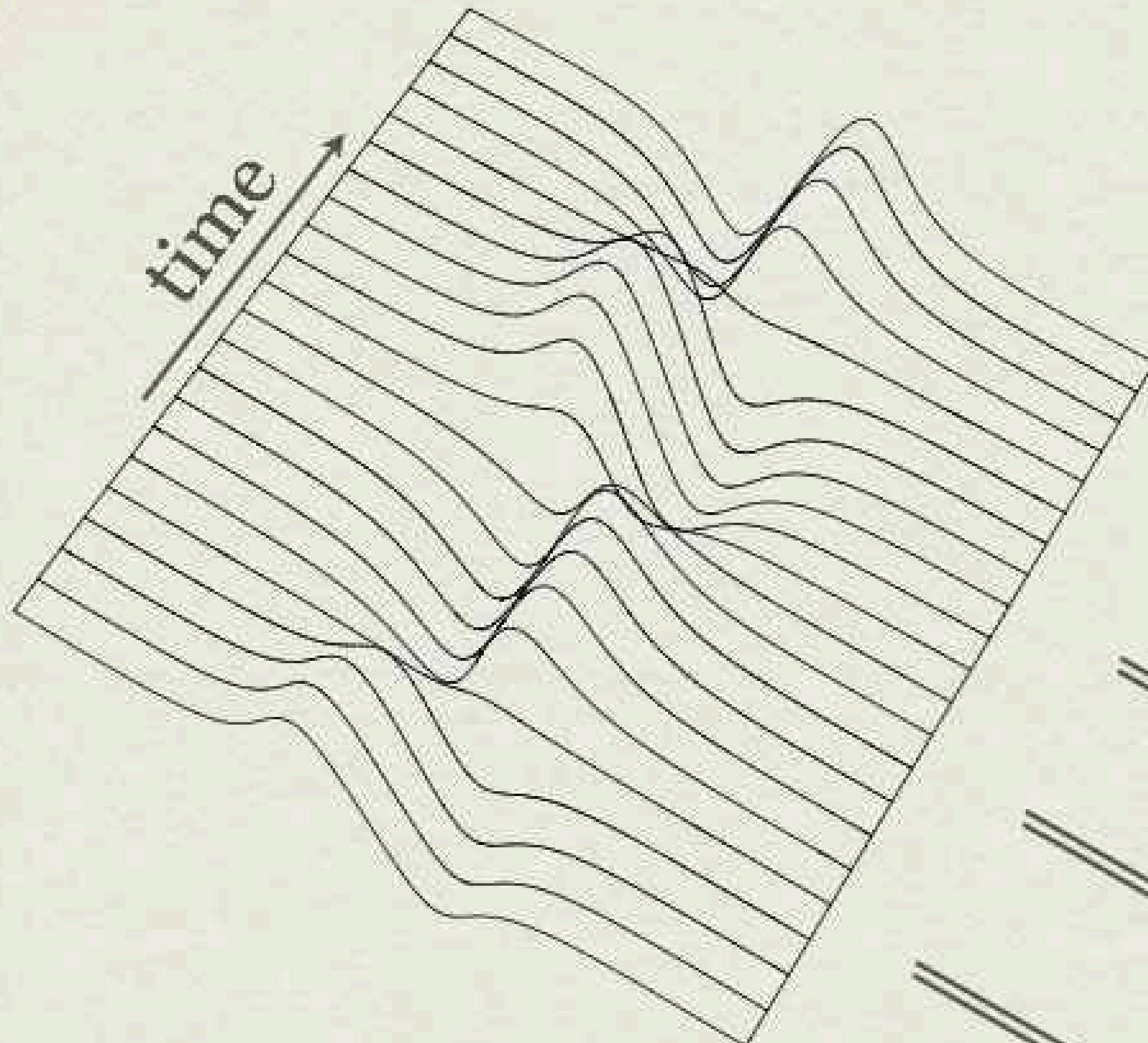


penetrate

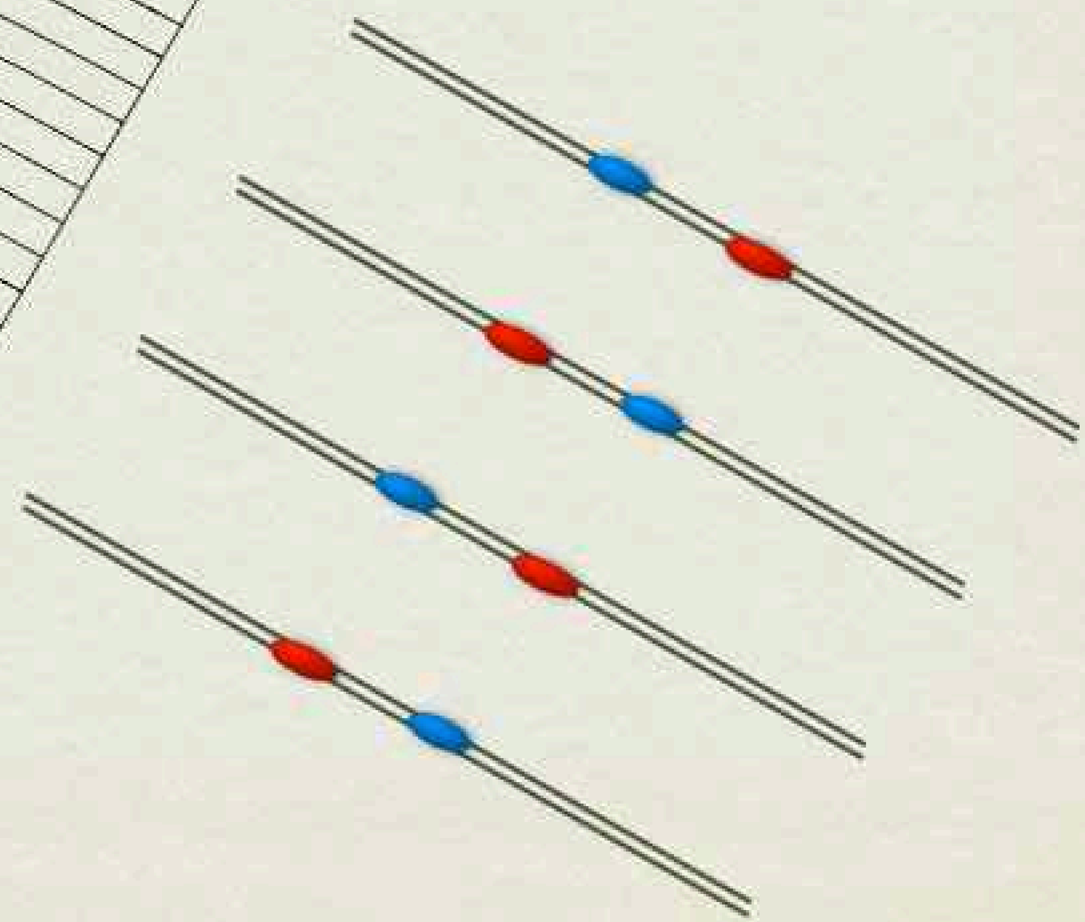



1
2
3
4
5

breather



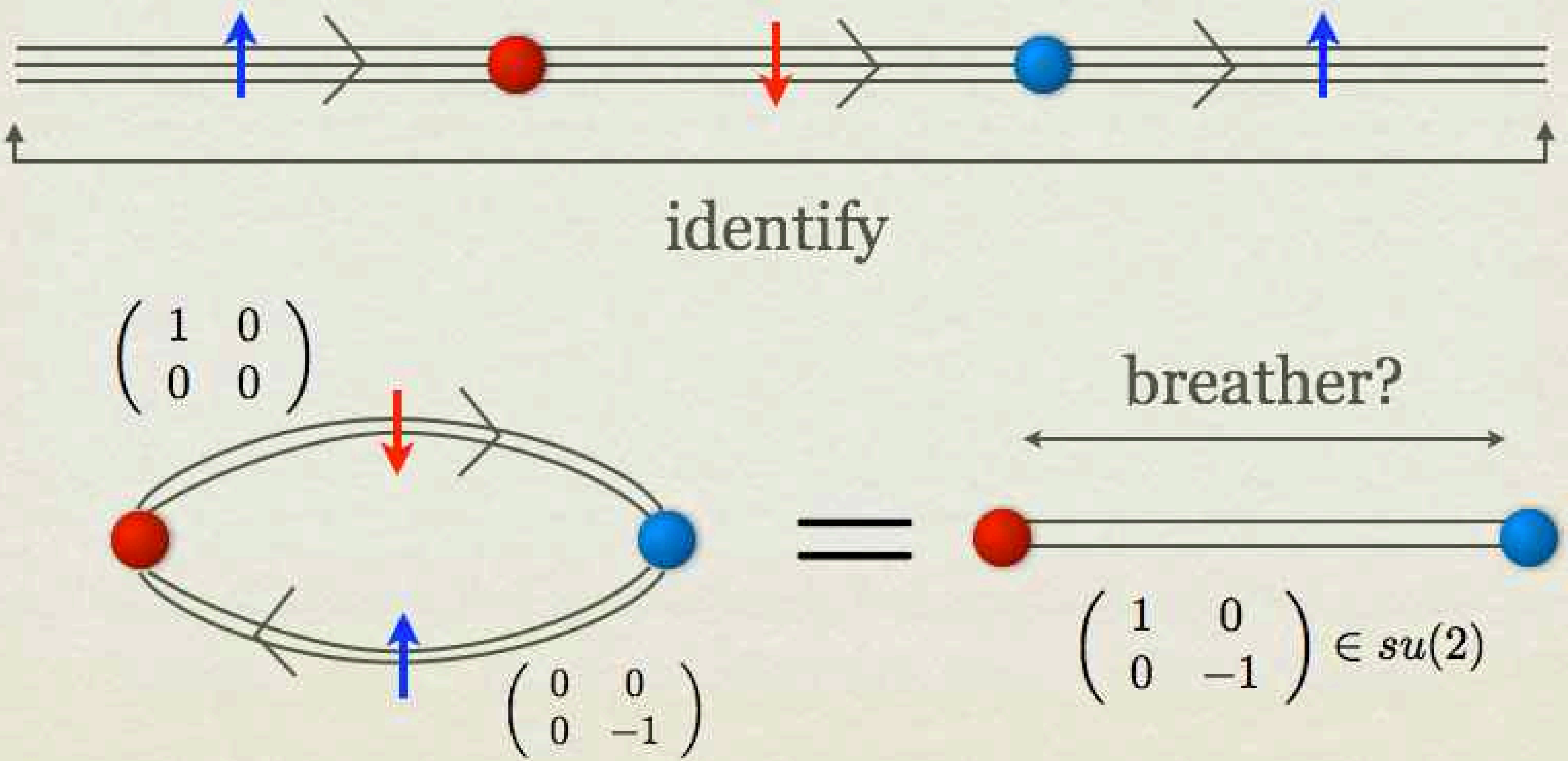
bound state



With an aid of vortex, analytical solutions of monopole dynamics are found!

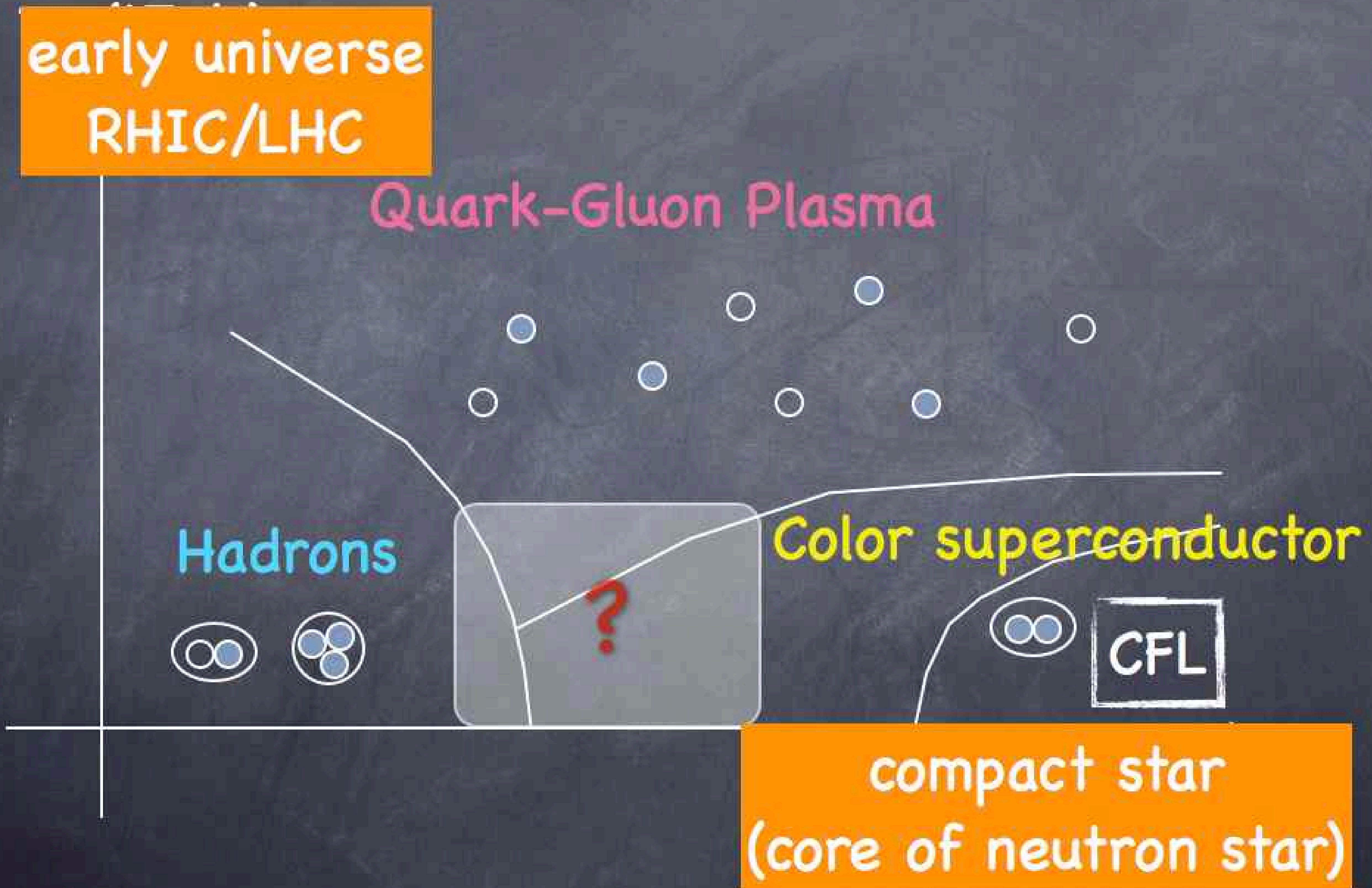
(No analytic solutions of monopole dynamics in coulomb phase)

Our solution indicates that stable mesonic particle exists!!



HIGH DENSITY QCD
COLOR SUPERCONDUCTOR

QCD phase diagram



Non-Abelian flux tube in real QCD

Asymptotically high baryon density $\left\{ \begin{array}{l} \text{weak coupling} \\ \text{diquark condensation} \\ \text{color superconductor} \\ \text{CFL phase (3 flavor)} \end{array} \right.$

[Alford-Rajagopal-Wilczek (`99)]

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R}$$

color-super　　superfluid

vortex solution: [Balachandran-Digal-Matsuura (`06)]

non-Abelian orientation: [Matsuura-Nakano-Nitta(`08)]

vortex worldsheet theory: [Eto-Nakano-Nitta(`09)]

(in)stability of flux tube: [Eto-Nitta-Yamamoto(`09)]

monopole : [Eto-Nitta-Yamamoto(`11)]

$$\frac{SU(3)_{c+L+R}}{U(2)_{c+L+R}} \simeq \mathbb{C}P^2$$

(Many other progresses done by Nitta-san and collaborators)

CONCLUSION

- relation between confinement and topological solitons
- SUSY: Abelian superconductor
- SUSY: non-Abelian superconductor
- monopole dynamics from vortex
- color superconductor

I emphasize that these topics have been developed via deep understanding of the recently found non-Abelian vortex!

BACK UP

Duality via vortex-string

A) BPS spectrum of d=3+1 N=2 SU(N) SYM

|| [N.Dorey (`98)]

B) BPS spectrum of d=1+1 N=(2,2) CP(N-1)

Why?

B) is effective theory on vortex string in d=3+1!

[Hanany-Tong (`04)]

monopole in A) = kink in B)