摂動QCDによる
クォーク・反クォーク対系の理解の進展

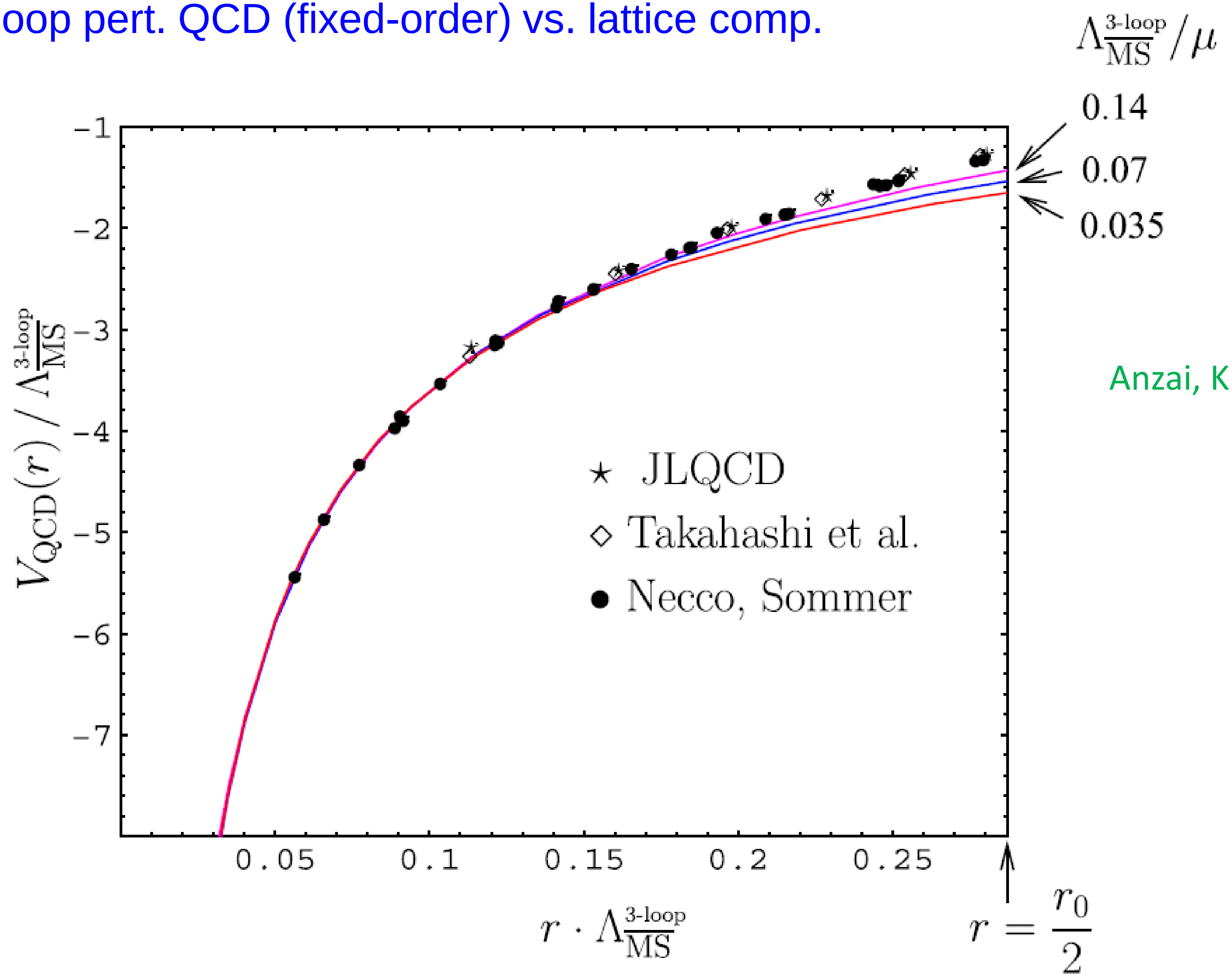
［Understanding Heavy Quark-AntiQuark System
by Perturbative QCD］

隅野行成

東北大学

Current Status of Static Potential

3-loop pert. QCD (fixed-order) vs. lattice comp.



Anzai, Kiyo, YS

☆Plan of Talk

1. Before 1998: Theoretical problem

   IR renomalon

2. Around 1998: Drastic improvement

   Discovery of cancellation of renormalons

   Interpretation

3. After 1998: Applications

   Spectroscopy

   Decays

   Determinations of m_b, m_c (m_t)

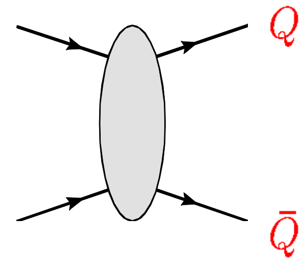
   Determination of α_s

   Gluon config. inside quarkonium

   Casimir scaling violation for static potential

   ⋮

1. Before 1998: Theoretical problem



Non-rel. Hamiltonian determined by matching to pert. QCD

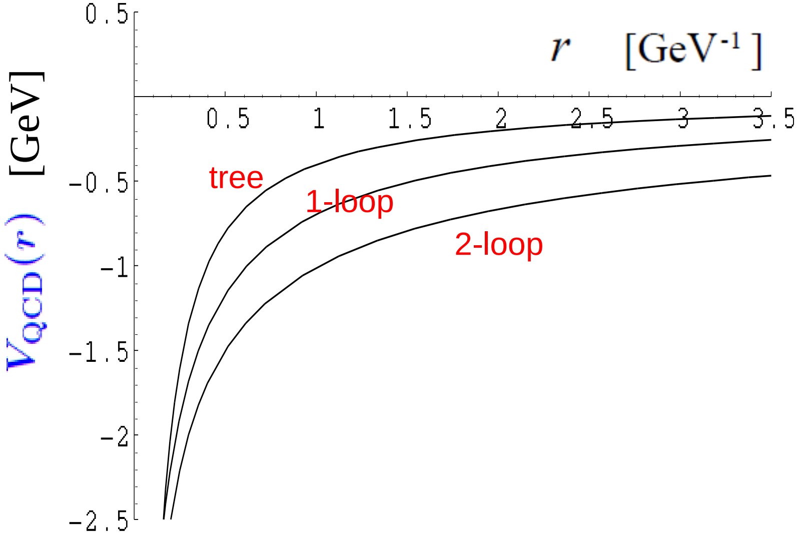
$$\hat{H}_0 = \frac{\vec{p}^{\,2}}{m} \underline{- C_F \frac{\alpha_S}{r}},$$

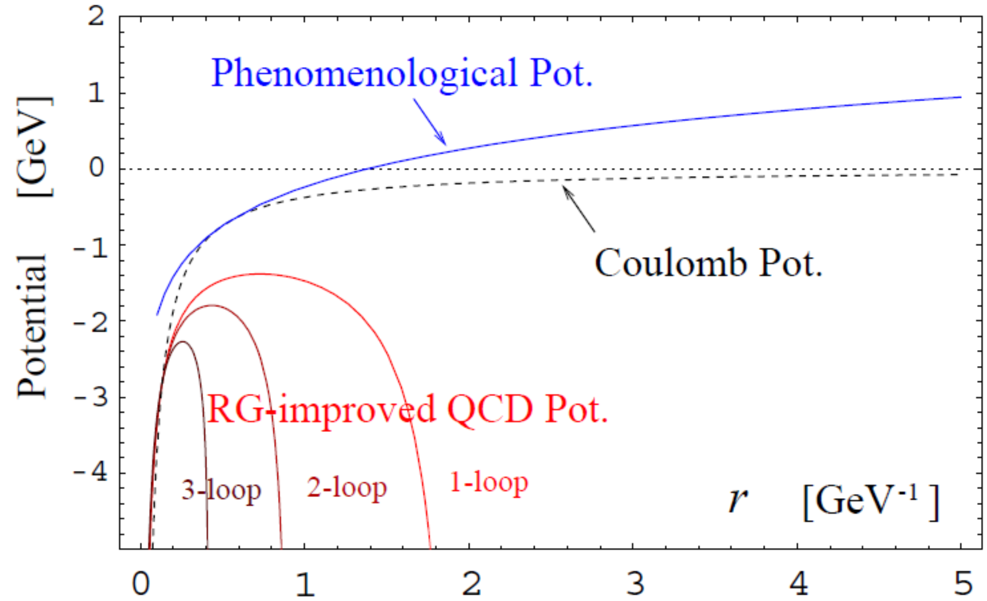
$$\hat{H}_1 = \underline{- C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right) \cdot \left\{\beta_0 \log(\mu'^2 r^2) + a_1\right\}},$$

$$\begin{aligned}\hat{H}_2 = & -\frac{\vec{p}^{\,4}}{4m^3} \underline{- C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right)^2 \cdot \left\{\beta_0^2 \left[\log^2(\mu'^2 r^2) + \frac{\pi^2}{3}\right] + (\beta_1 + 2\beta_0 a_1) \log(\mu'^2 r^2) + a_2\right\}} \\ & + \frac{\pi C_F \alpha_S}{m^2} \delta^3(\vec{r}) + \frac{3 C_F \alpha_S}{2m^2 r^3} \vec{L} \cdot \vec{S} - \frac{C_F \alpha_S}{2m^2 r}\left(\vec{p}^{\,2} + \frac{1}{r^2} r_i r_j p_j p_i\right) - \frac{C_A C_F \alpha_S^2}{2 m r^2} \\ & - \frac{C_F \alpha_S}{2m^2}\left\{\frac{S^2}{r^3} - 3\frac{(\vec{S}\cdot\vec{r})^2}{r^5} - \frac{4\pi}{3}(2S^2 - 3)\delta^3(\vec{r})\right\},\end{aligned}$$

Titard, Yndurain; Pineda Yndurain

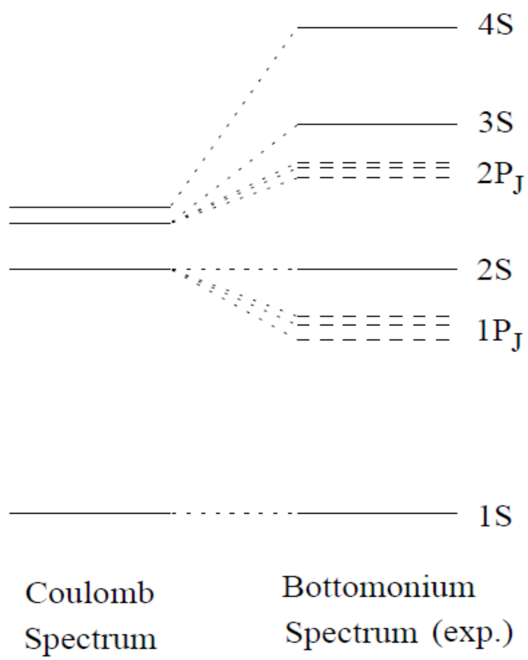
- $\Upsilon(1S): \quad M_{\Upsilon(1S)} = \underset{\mathcal{O}(\alpha_s^0)}{\overset{2m_{b,pole}}{9.94}} - \underset{\mathcal{O}(\alpha_s^2)}{0.17} - \underset{\mathcal{O}(\alpha_s^3)}{0.20} - \underset{\mathcal{O}(\alpha_s^4)}{0.30} \ \text{GeV}$





Folklore: pert. $\sim -C_F \dfrac{\alpha_s}{r}$, non-pert. $\sim K\, r$

$$V_{\rm QCD}(r) \simeq -C_F \frac{\alpha_s}{r} + K\, r$$



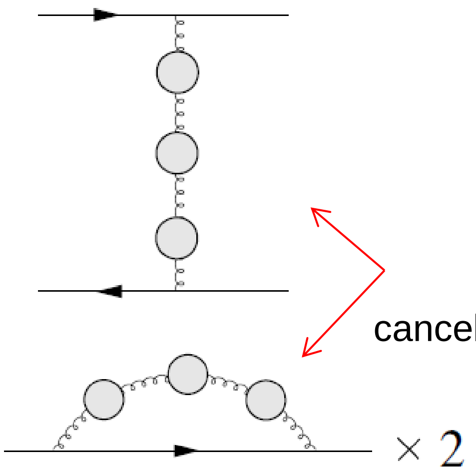
Accuracy of perturbative predictions for the QCD potential improved drastically around year 1998.

Pineda
Hoang,Smith,Stelzer,Willenbrock
Beneke

If we re-express the quark pole mass ($m_{\rm pole}$)
by the $\overline{\rm MS}$ mass ($m_{\overline{\rm MS}}$), IR renormalons cancel
in $E_{\rm tot}(r) = 2m_{\rm pole} + V_{\rm QCD}(r)$.

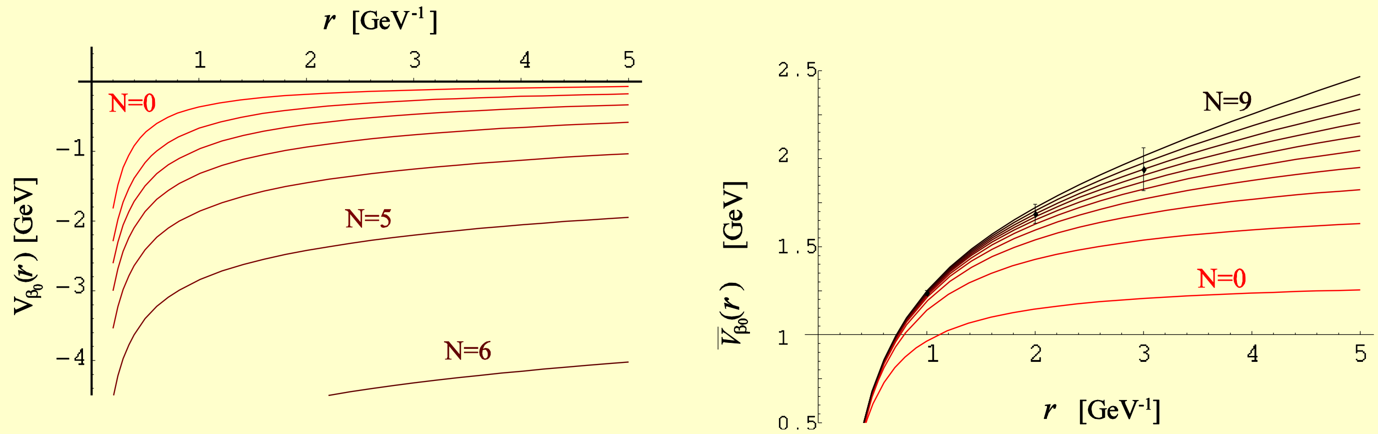
$$V_{\rm LL}(r) \;=\; -\int \frac{d^3\vec{q}}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}}\, C_F\, \frac{4\pi\alpha_{1L}(q)}{q^2}$$

$$m_{\rm pole} \simeq m_{\overline{\rm MS}}(\mu) + \frac{1}{2}\int_{q<\mu} \frac{d^3\vec{q}}{(2\pi)^3}\, C_F\, \frac{4\pi\alpha_{1L}(q)}{q^2}$$



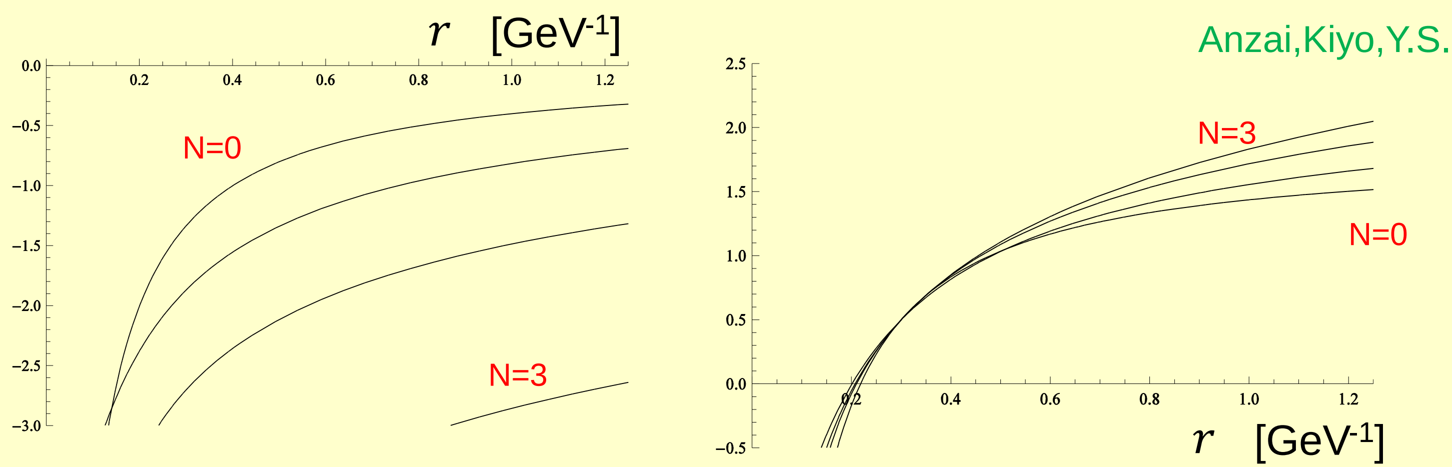
Expanding $e^{i\vec{q}\cdot\vec{r}} = \underline{1} + i\vec{q}\cdot\vec{r} + \frac{1}{2}(i\vec{q}\cdot\vec{r})^2 + \cdots$ for small q ,
the leading renormalons cancel . ⇨ much more convergent series

Residual renormalon: $\Lambda \times \left\langle (\vec{q}\cdot\vec{r})^2 \right\rangle \sim \Lambda \times \left(\Lambda\, r\right)^2 \ll \Lambda$

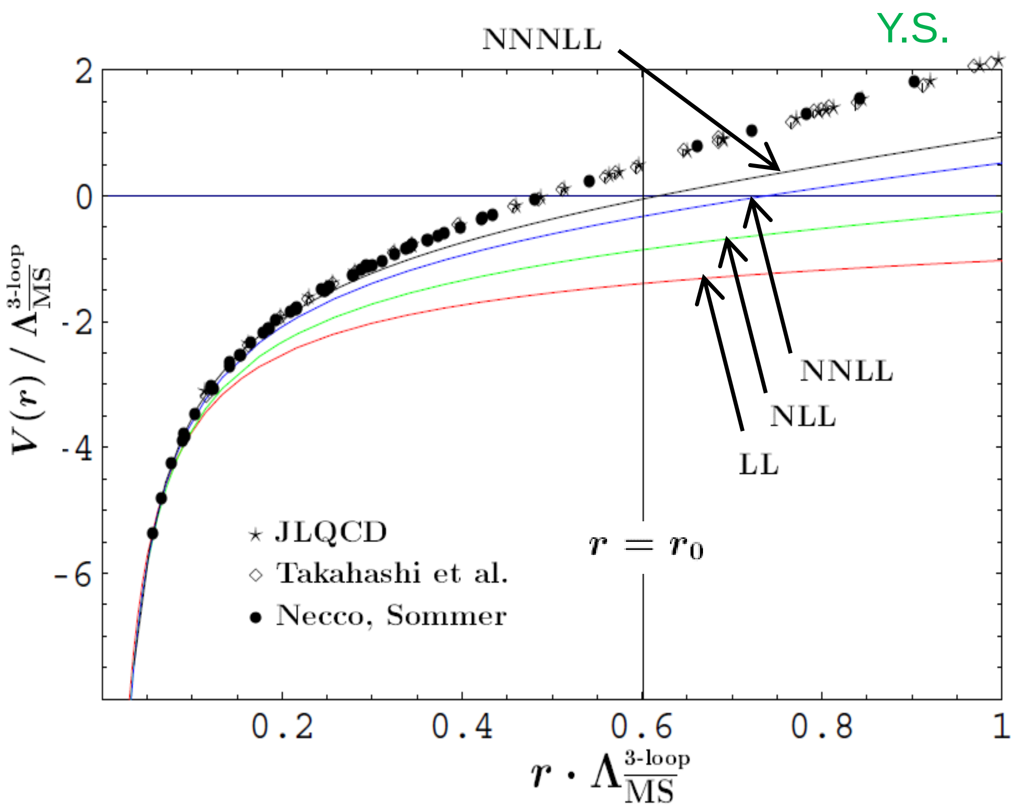


Leading log approximation

Anzai,Kiyo,Y.S.



Exact pert. potential up to 3 loops



- Agreement with lattice comp. extends to larger r at higher orders.
- Stable prediction up to larger r with higher-order corrections.

Folklore ruled out

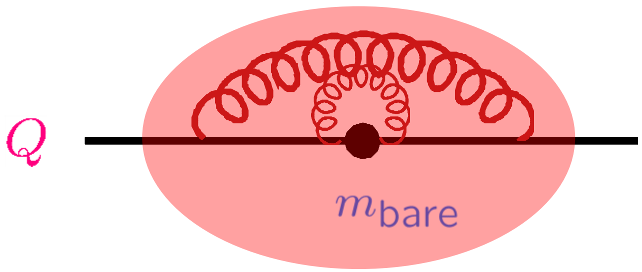
pert. $\sim -C_F \frac{\alpha_s}{r}$, non-pert. $\sim K\, r$

$$V_{\rm QCD}(r) \simeq -C_F \frac{\alpha_s}{r} + K\, r$$

- $\Upsilon(1S)$: $M_{\Upsilon(1S)} = 9.94 - 0.17 - 0.20 - 0.30$ GeV (Pole-mass scheme)
  $= 8.41 + 0.72 + 0.15 + 0.015 - 0.008$ GeV ($\overline{\rm MS}$-scheme)

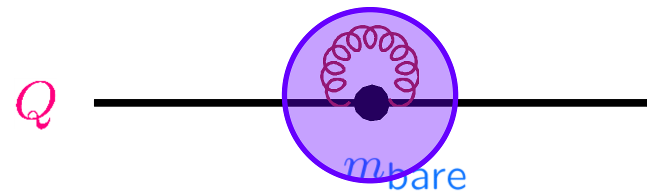
Pole mass m_{pole}

$0 < \lambda_g < \infty$



$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

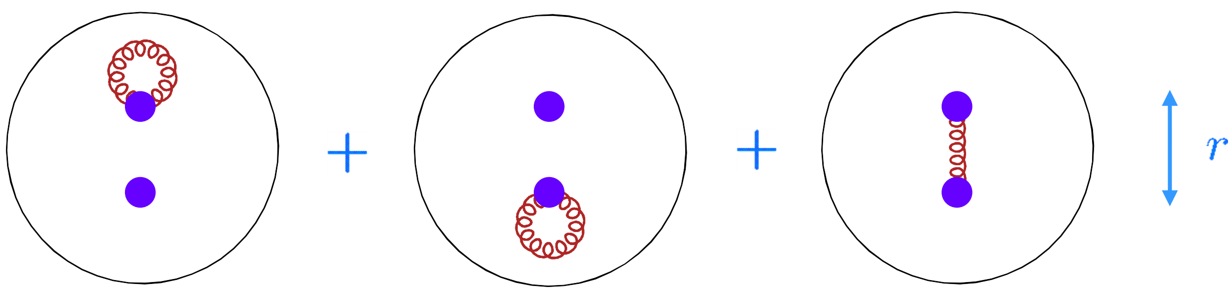
$0 < \lambda_g < 1/\overline{m}$



Computation of spectrum of Heavy Quarkonium ($Q\bar{Q}$ boundstate)

Using $\overline{\text{MS}}$ mass $\quad 2m_{\text{pole}} = 2\overline{m}\,(1 + c_1\,\alpha_S + c_2\,\alpha_S^2 + c_3\,\alpha_S^3 + \cdots)$

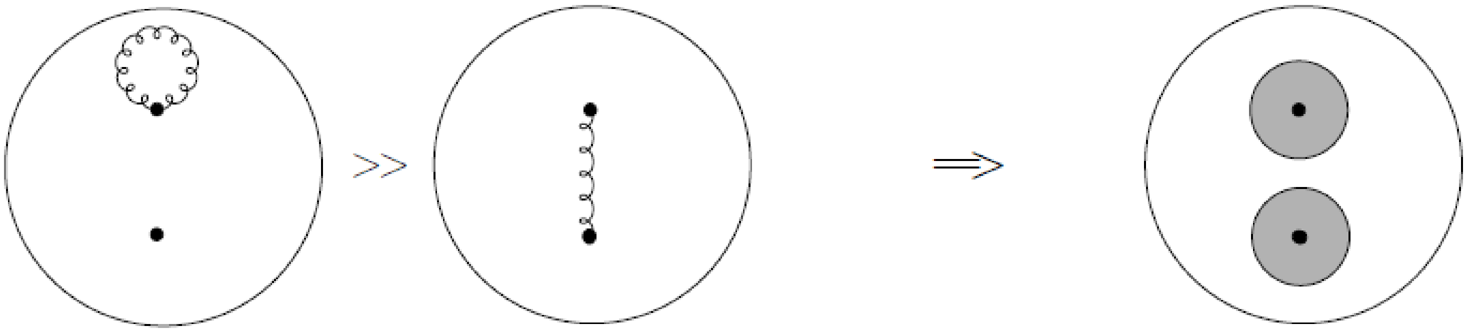
$E_{\text{tot}} =$

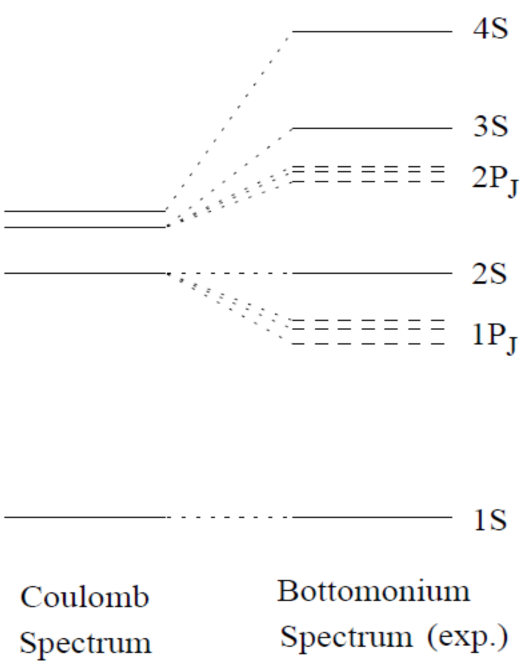


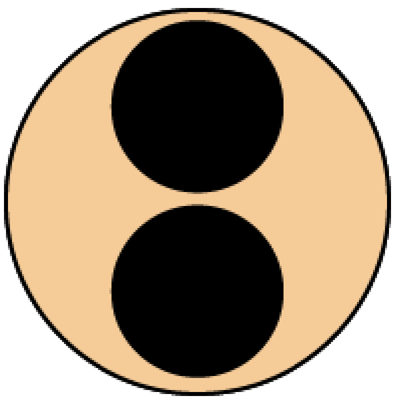
IR gluons $\lambda_g \gg r$ decouple ⇨ much more convergent series

Rapid growth of masses of excited states originates from rapid growth of self-energies of Q & $\overline{Q}$ due to IR gluons.

Brambilla, Y.S., Vairo







$$E_X \approx 2m_b^{\overline{MS}}(\mu) + \int_0^\mu dq\, f_X(q)\alpha_s(q)$$

Brambilla,YS,Vairo
Recksiegel,YS

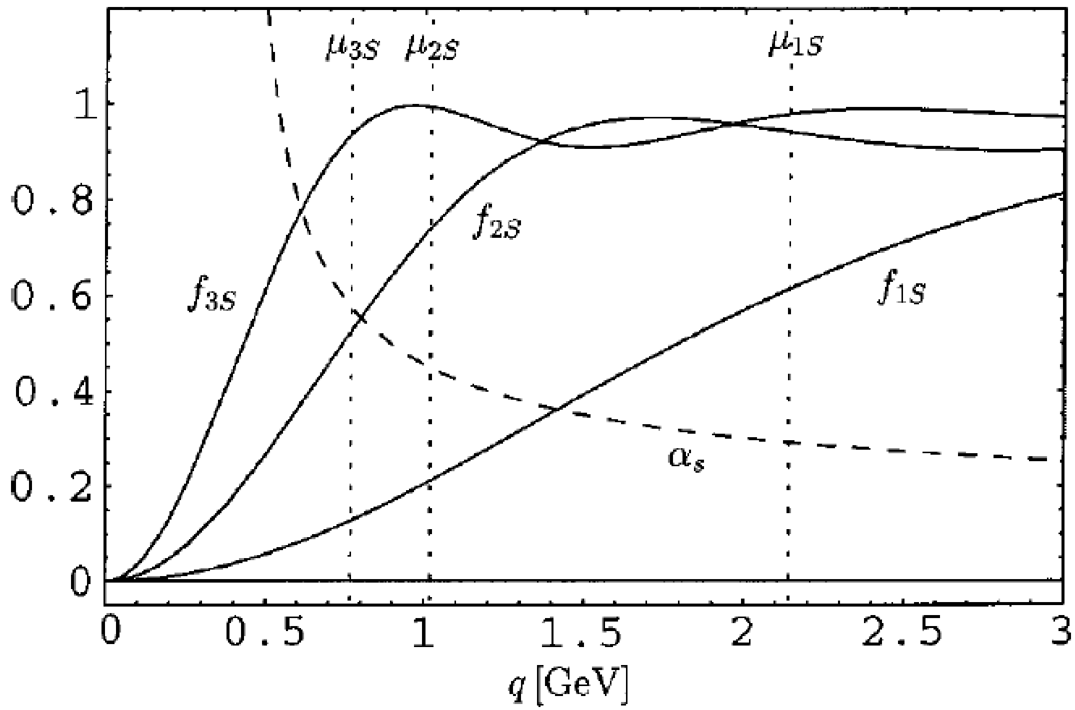
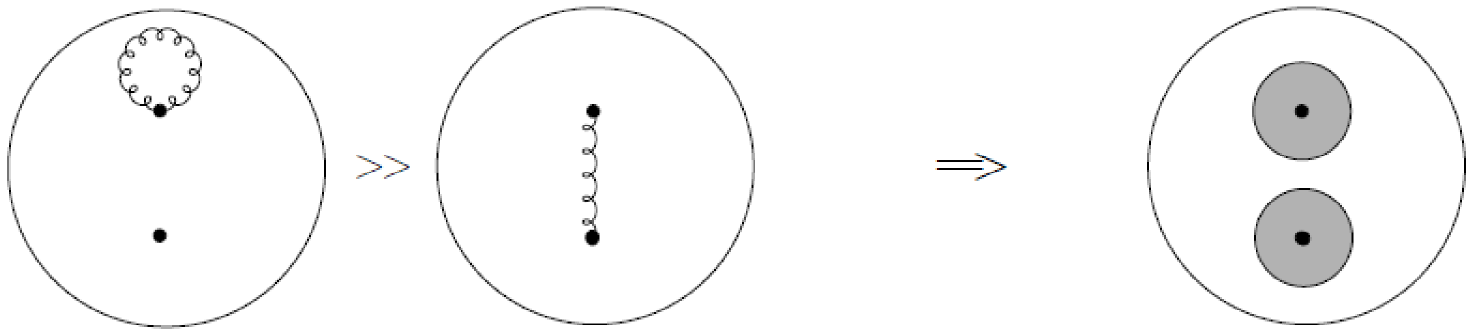
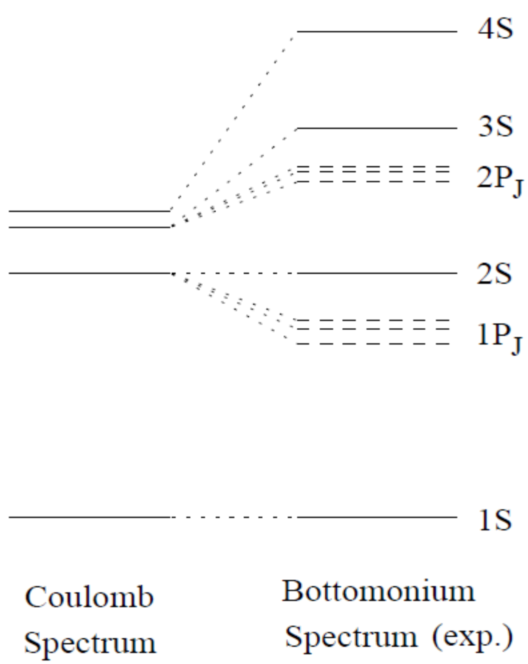


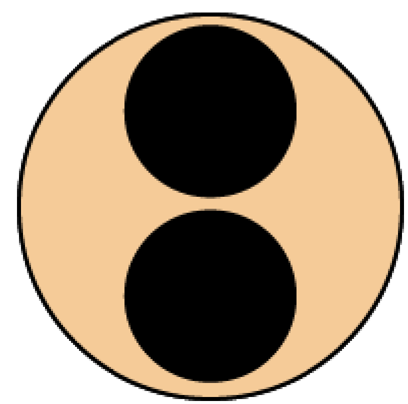
FIG. 5. Support functions for the S states. The solid curves show the support functions as defined in Eq. (19); for comparison of the relevant scales, $\alpha_s^{(4)}(\mu)$ is also plotted (dashed curve). Since the analysis that we advocate in this work does not attribute scales to the individual states, the scales indicated by the dotted lines are taken from [3], Table II.

Rapid growth of masses of excited states originates from rapid growth of self-energies of Q & $\bar{Q}$ due to IR gluons.

Brambilla, Y.S., Vairo







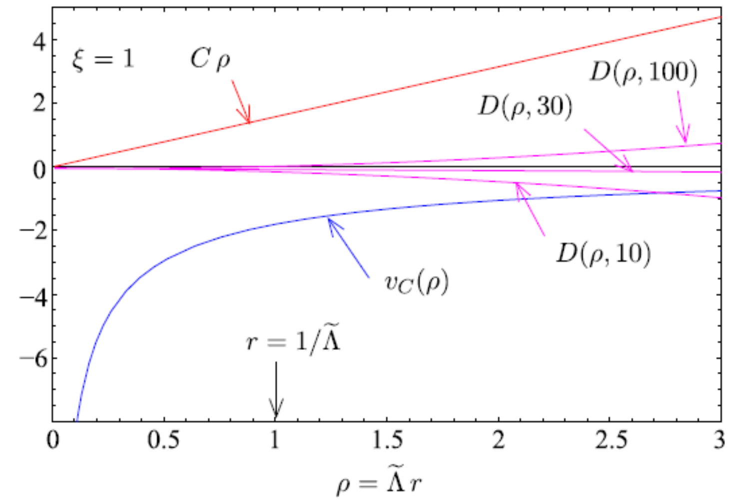
Reminiscent of
constituent quark mass picture

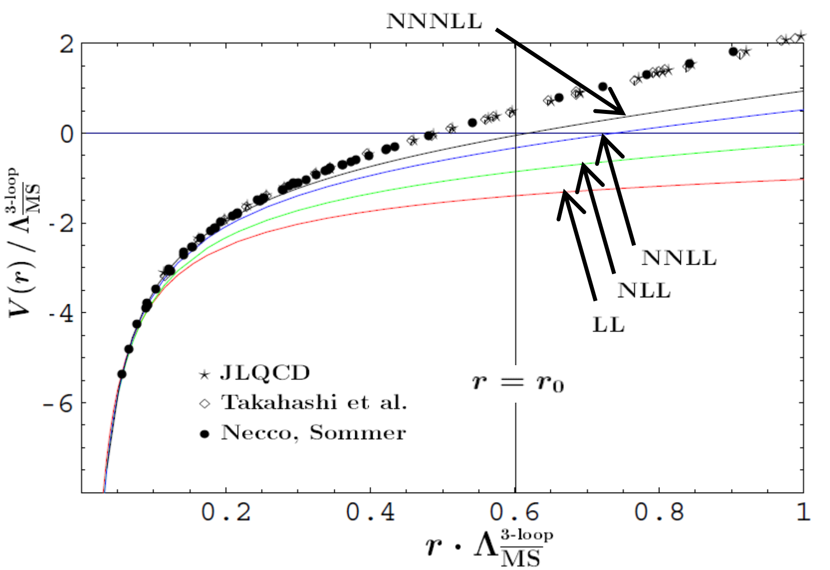
A ‘Coulomb+Linear potential’ is obtained by resummation of logs: YS

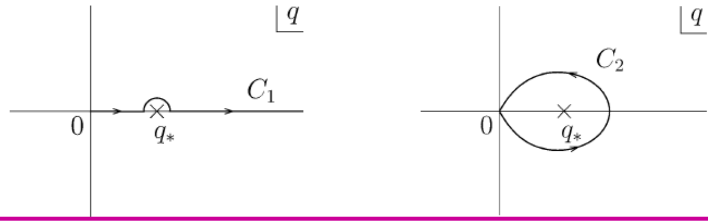
$$\begin{array}{ccccc} & r^{-1} & r^{0} & r^{1} & r^{2} \\ V_{\rm QCD}(r) = & \underline{V_C(r)} & + \text{const.} + & \underline{\sigma r} & + \mathcal{O}(\Lambda^3 r^2) \end{array}$$

Free of IR renormalons

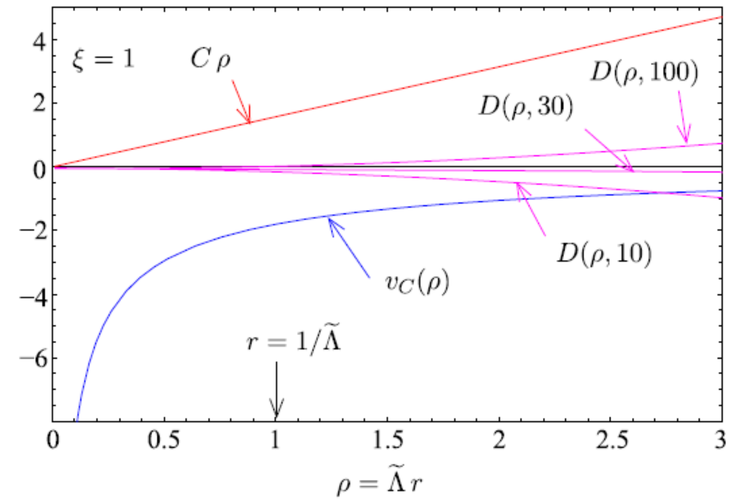
Pert. prediction valid at $r \lesssim \Lambda_{\rm QCD}^{-1}$







A 'Coulomb+Linear potential' is obtained by resummation of logs: YS



$$V_{\rm QCD}(r) = \underline{V_C(r)} + \text{const.} + \underline{\sigma r} + \mathcal{O}(\Lambda^3 r^2)$$

- $$V_C(r) = -\frac{4\pi C_F}{\beta_0 r} - \frac{2C_F}{\pi}\,{\rm Im}\int_{C_1} dq\,\frac{e^{iqr}}{qr}\,\alpha_V(q)$$

Coulombic pot.
(with log corr. at short-dist.)

$$= \begin{cases} -\dfrac{2\pi C_F}{\beta_0}\,\dfrac{1}{r|\log(\Lambda_{\overline{\rm MS}} r)|}\left[1 - \dfrac{\delta}{2}\,\dfrac{\log|\log(\Lambda_{\overline{\rm MS}} r)|}{|\log(\Lambda_{\overline{\rm MS}} r)|}\right], & r \to 0, \\ -\dfrac{4\pi C_F}{\beta_0 r}, & r \to \infty, \end{cases}$$

- $$\sigma = \frac{C_F}{2\pi i}\int_{C_2} dq\, q\, \alpha_V(q)$$

Coefficient of linear pot.

$$\alpha_V(q) = \alpha_s(q)\sum_{n=0}^{N} a_n \left(\frac{\alpha_s(q)}{4\pi}\right)^n$$

$N = 0, 1, 2, 3, \cdots$ for
LL, NLL, NNLL, NNNLL, $\cdots$.

$$\text{e.g.}\quad \sigma_{\rm NLL} = \frac{2\pi C_F}{\beta_0}\left(\Lambda_{\overline{\rm MS}}^{\text{2-loop}}\right)^2 \frac{e^{-\delta}}{\Gamma(1+\delta)}\left[1 + \frac{a_1}{\beta_0}\,\delta^{-1-\delta}\,e^{\delta}\,\gamma(1+\delta,\delta)\right]$$

$$\delta = \beta_1/\beta_0^2$$

3. After 1998: Many Applications

Spectroscopy

Decays

Determinations of m_b, m_c (m_t)

Determination of α_s

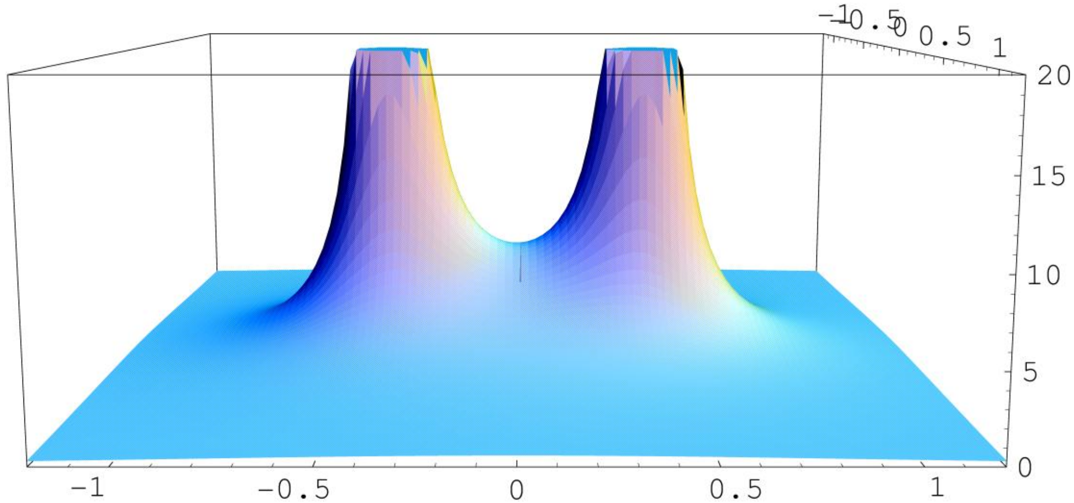
Gluon config. inside quarkonium

Casimir scaling violation for static potential

$\vdots$

☆Action density surrounding heavy quarks

3D plot of $\sqrt{\rho_{\beta_0}(\vec{x},\vec{r})}$ $R = 0.8\ \tilde{\Lambda}^{-1} \approx 0.4$ fm



Color electric field ($\propto\sqrt{\rho(\vec{x},\vec{r})}$) between Q and $\bar{Q}$ is stronger than pure Coulomb field $\Longleftarrow$ $\nabla V_L(R)$

Domain of color electric field (action density) grows isotropically and scales as $\sim R^3$

Action density distributions corresponding to "Coulomb" and linear potentials:

Casimir scaling hypothesis

····· supported by lattice measurements

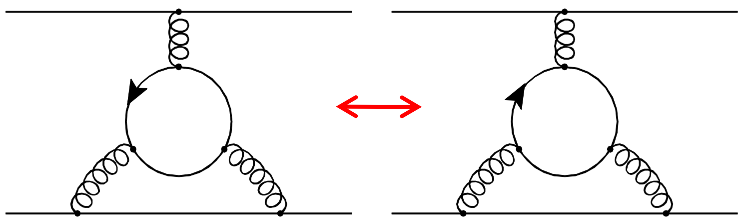
Markum,Faber
Campbell,Jorysz,Michael
Deldar
Bali

$V_R(r) \propto C_R$

2nd Casimir op. for rep. R

cf. $V_R^{\rm tree}(r) = -C_R \dfrac{\alpha_s}{r}$

2-loop



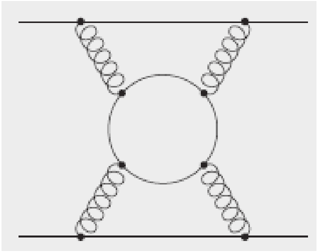
$d_R^{abc} d_F^{abc}$ cancel

$$\left(d_R^{a_1 \cdots a_n} = \frac{1}{n!} \mathrm{Tr}\left[T_R^{a_1} T_R^{a_2} \cdots T_R^{a_n} + (\text{all permutations}) \right] \right)$$

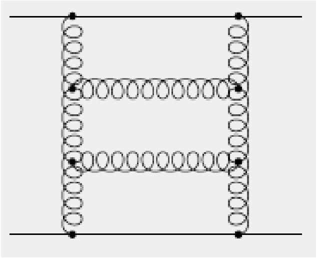
$d_A^{abc} = 0$ by $(T_A^a)^T = -T_A^a$

protected by C-inv.

3-loop



$d_R^{abcd} d_F^{abcd}$



$d_R^{abcd} d_A^{abcd}$

Anzai, Kiyo, YS

⇨ Casimir scaling violation

Tiny violation predicted, compatible with current lattice data.

☆Summary

1. Before 1998: Theoretical problem

   IR renomalon

2. Around 1998: Drastic improvement

   Discovery of cancellation of renormalons

   Interpretation, a linear rise at $r \lesssim \Lambda_{\rm QCD}^{-1}$

3. After 1998: Applications (besides theoretical developments)

   Spectroscopy

   Decays

   Determinations of m_b, m_c (m_t)

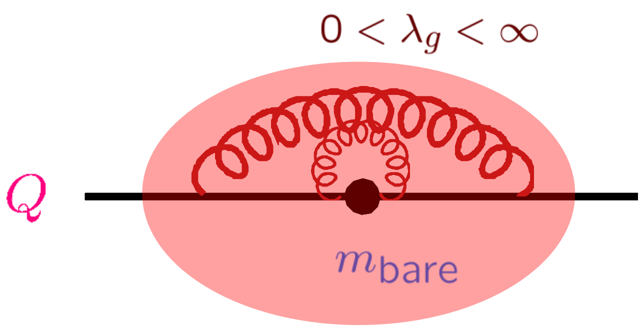
   Determination of α_s

   Gluon config. inside quarkonium

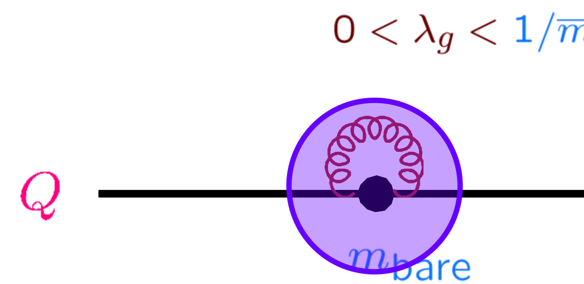
   Casimir scaling violation for static potential

   ⋮

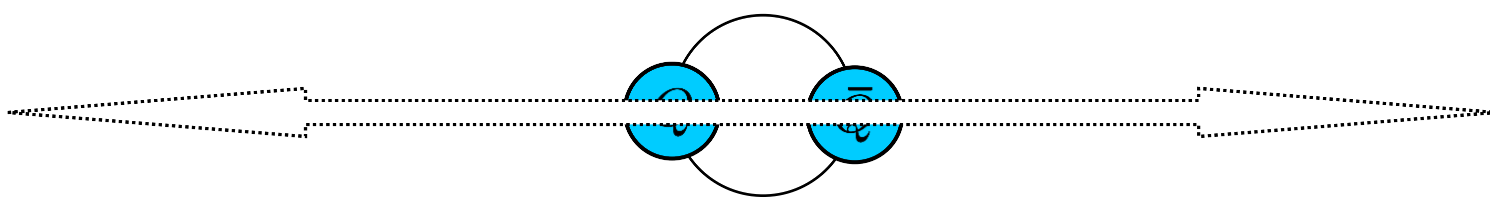
Pole mass m_{pole}



$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$



Computation of spectrum of Heavy Quarkonium ($Q\bar{Q}$ boundstate)



$$\left.\begin{array}{l}\text{Free } Q \text{ and } \bar{Q}: \\ E_{\text{tot}} = 2m_{\text{pole}} - E_{\text{bin}}\end{array}\right\} \text{not well-defined}$$

Poorly convergent perturbative series

Application to quarkonium spectroscopy and determination of α_s, m_b, m_c.

- Global level structure of bottomonium is reproduced. Brambilla, Y.S., Vairo; Recksiegel, Y.S

- Fine and hyperfine splittings of charmonium/bottomonium reproduced.

  Two exceptions in ~2003: Recksiegel, Y.S.; Kniehl, Penin, Pineda, Smirnov, Steinhauser

  charmonium hyperfine splitting $\Psi(2S)-\eta_c(2S)$

  bottomonium hyperfine splitting $\Upsilon(1S)-\eta_b(1S)$

  Solved in favor of pert. QCD predictions.

- Determination of bottom and charm quark $\overline{\text{MS}}$ masses:

$$\overline{m}_b(\overline{m}_b) = 4190 \pm 30 \text{ MeV}$$
$$\overline{m}_c(\overline{m}_c) = 1243 \pm 100 \text{ MeV}$$

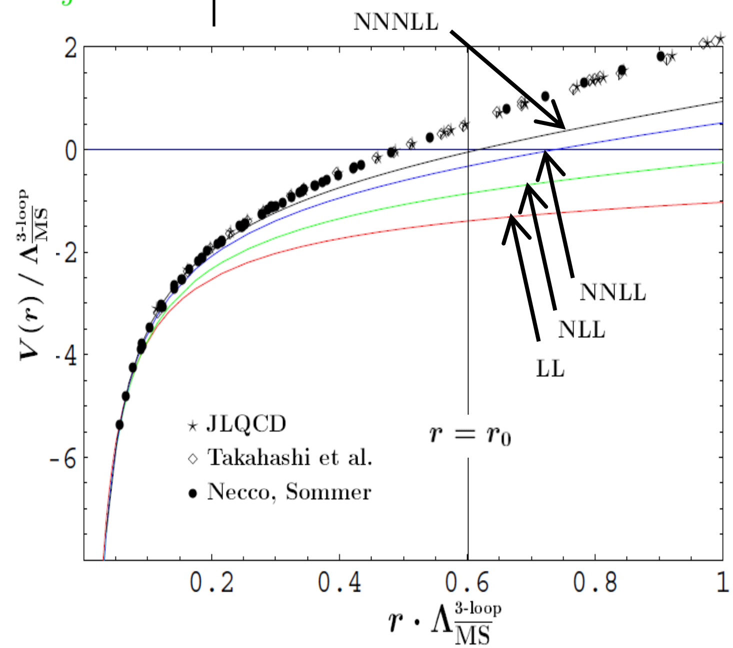
Brambilla, Y.S., Vairo

- Relation between lattice α_s and $\overline{\text{MS}}$ α_s being accurately measured

  (realistic precision determination in near future) Y.S.

  Brambilla, Petreczky, Tormo, Soto, Vairo

Bottomonium spectrum up to $n = 3$ states



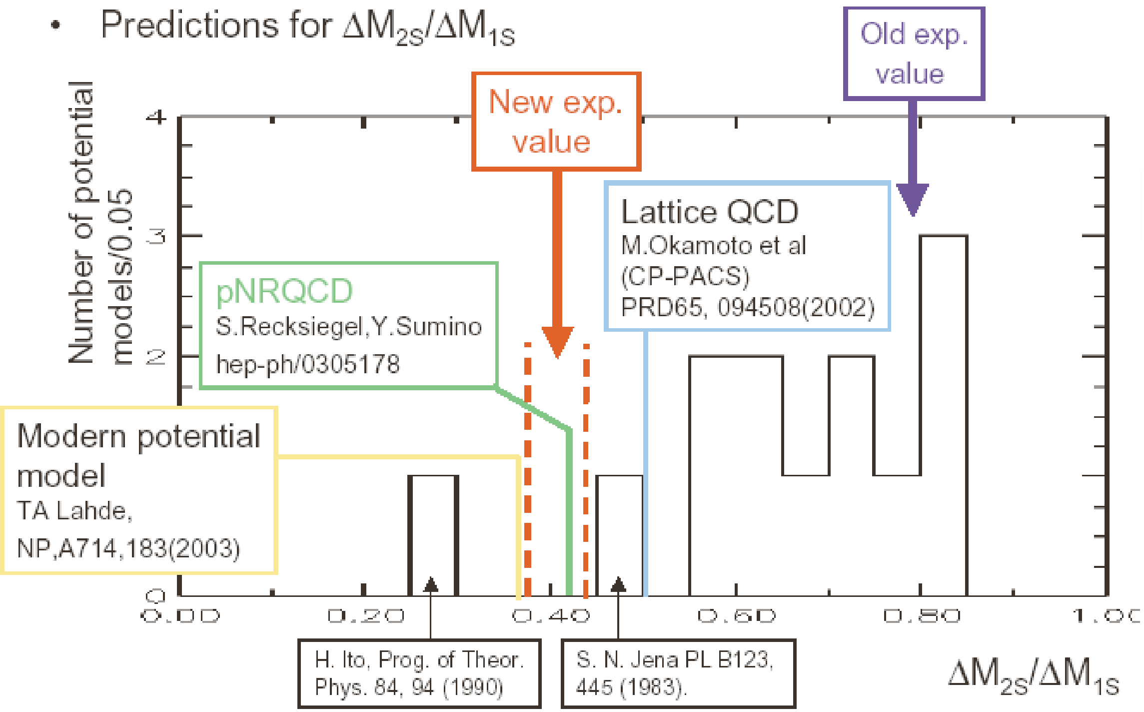
* Pert. QCD prediction including full $\mathcal{O}(\alpha_S^4 m)$ corrections to individual energy levels, as well as full $\mathcal{O}(\alpha_S^5 m)$ corrections to fine structure [$\alpha_S(M_Z) = 0.1181$].

Recksiegel, Y.S.

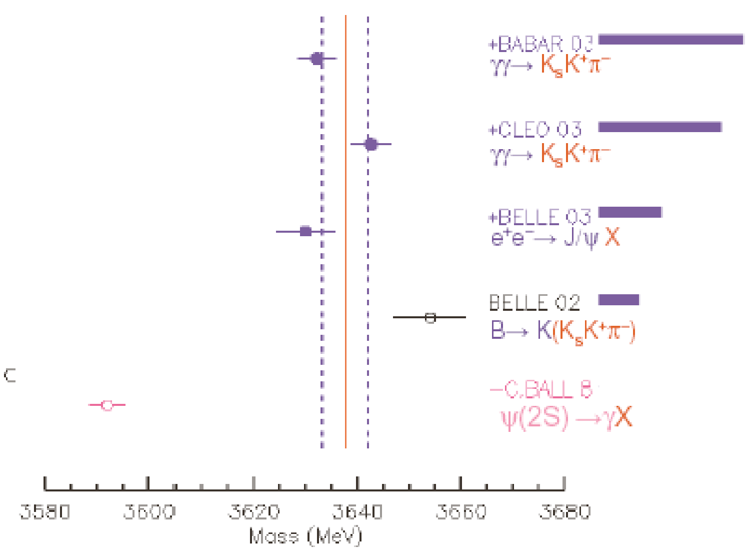
Slides from Skwarnicki's plenary talk at Lepton-Photon 2003

Predictions for hyperfine splitting ratio

- For 20 years theorists were exposed to the experimental value of ΔM_{2S}=M(ψ(2S))-M(η_c(2S)) which was wrong by a factor of 2
- Predictions for $\Delta M_{2S}/\Delta M_{1S}$



3637.7±4.4 MeV



CL=14% scale factor=1.3

New measurements of mass are consistent

Motivation for precision determinations of heavy quark masses

- Bottom quark
  - Constraints on $b-\tau$ mass ratio of SU(5) GUT models
  - Input param. for b physics: e.g. $\Gamma_b \propto m_b^5$ ⇨ LHC$_b$, Super-B factory

- Top quark
  - The only quark mass without $\overline{\text{MS}}$ mass in current PDG data.

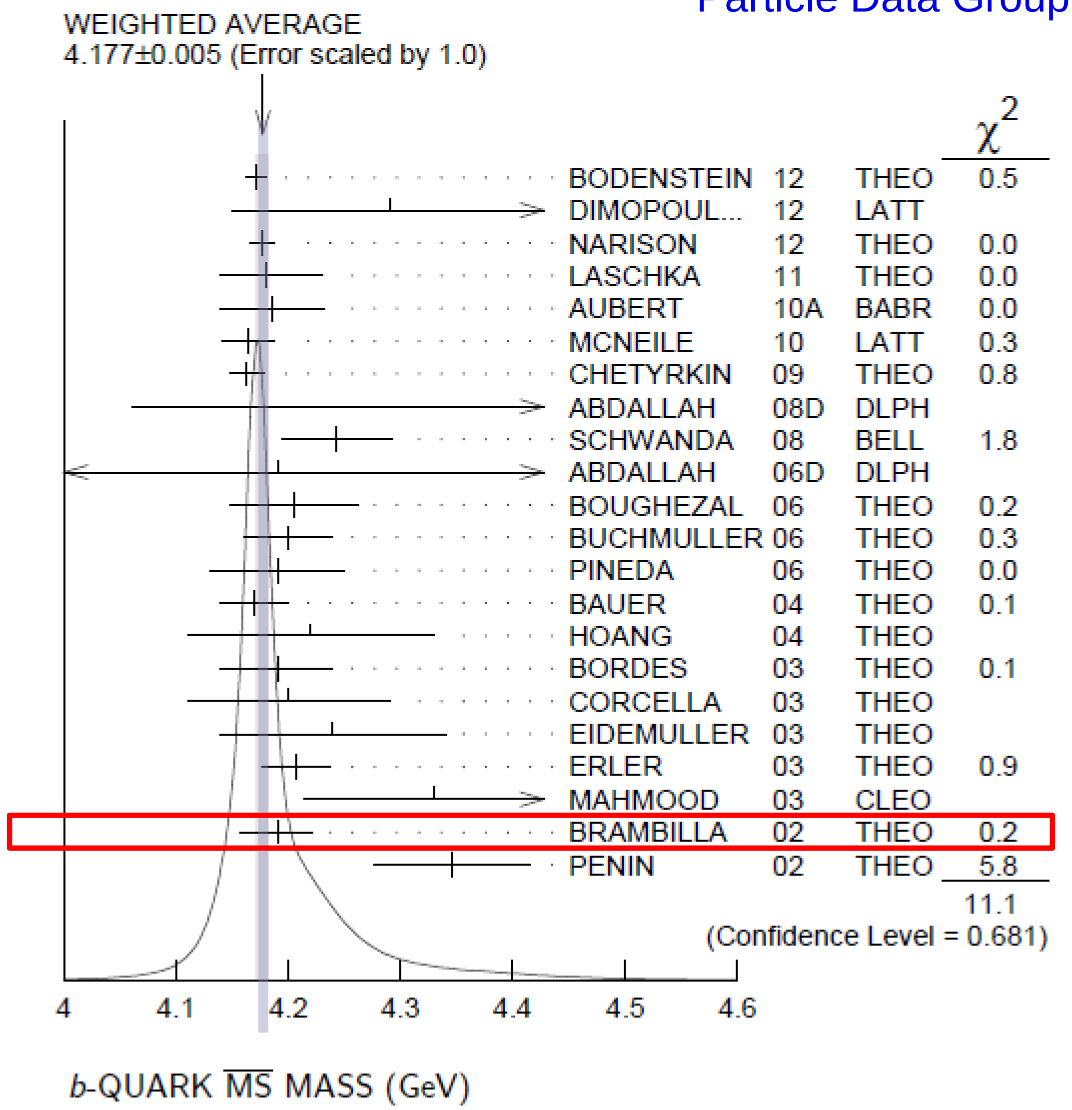
$$m_t = 173.5 \pm 0.6 \pm 0.8 \text{ GeV} \longleftarrow \text{What mass?}$$

  - Tests of Yukawa coupling at LHC and beyond.

$$\delta_t M_H^{\text{SM}} \propto m_t^2 \qquad \text{cf. } \Delta M_H \sim 0.1\text{–}0.2 \text{ GeV} \quad \text{LHC}$$

$$\delta_t M_H^{\text{MSSM}} \propto m_t^4 \qquad \sim 0.05 \text{ GeV} \quad \text{ILC}$$

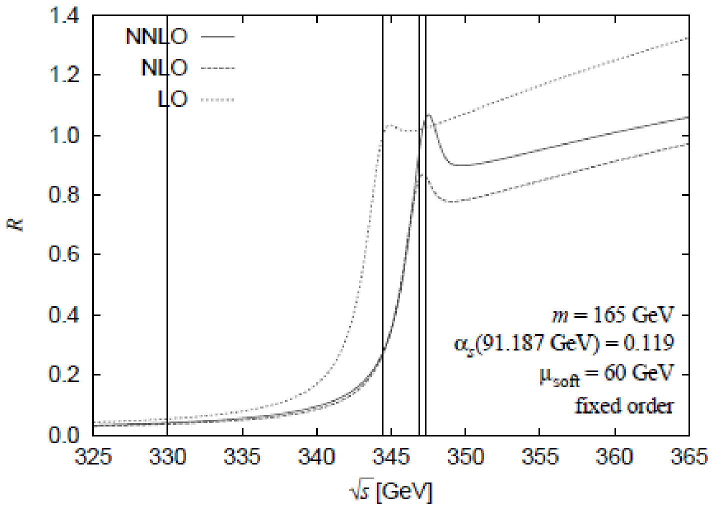
Particle Data Group 2012

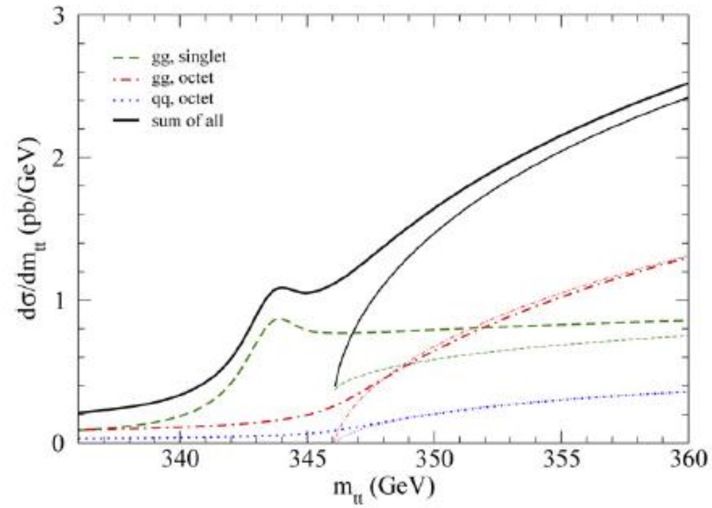


Prospects for precision determination of m_t from M_{tt}(1S)

Hagiwara,Y.S.,Yokoya
Kiyo, et al.

Hoang, et al.





$e^+e^- \to t\bar{t}$ in the threshold region
@ future Linear Collider

$pp \to t\bar{t}$ (threshold region)
@LHC

$\Delta\overline{m}_t \lesssim 100\ \mathrm{MeV}$

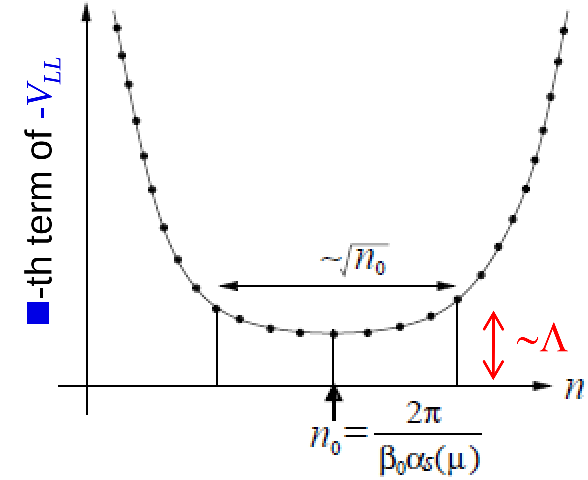
$\Delta\overline{m}_t$ significantly smaller than 1GeV?

Renormalon in the QCD potential

Aglietti, Ligeti

$$\alpha_{1L}(q) = \frac{\alpha_S(\mu)}{1+\frac{\beta_0\alpha_S(\mu)}{4\pi}\log\left(\frac{q^2}{\mu^2}\right)} = \frac{4\pi/\beta_0}{\log\left(\frac{q^2}{\Lambda^2}\right)}$$

$$\Lambda \equiv \mu\ \exp\left[-\frac{2\pi}{\beta_0\alpha_S(\mu)}\right]$$



$$\boxed{V_{LL}(r) = -\int\frac{d^3\vec{q}}{(2\pi)^3}\,e^{i\vec{q}\cdot\vec{r}}\,C_F\,\frac{4\pi\alpha_{1L}(q)}{q^2}}$$

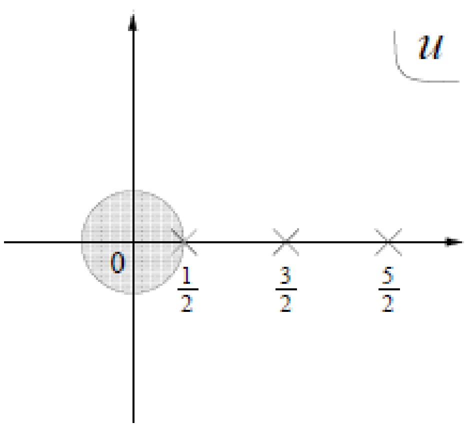
ill defined

$$= -C_F\,4\pi\alpha_S(\mu)\sum_{n=0}^{\infty}\int\frac{d^3\vec{q}}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}\left\{-\frac{\beta_0\alpha_S(\mu)}{4\pi}\log\left(\frac{q^2}{\mu^2}\right)\right\}^n$$

$$= -C_F\,4\pi\alpha_S(\mu)\sum_{n=0}^{\infty}\left\{\frac{\beta_0\alpha_S(\mu)}{4\pi}\right\}^n f_n(r,\mu)\times n!$$

$$F(r,\mu;u) \equiv \int\frac{d^3\vec{q}}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}\left(\frac{\mu^2}{q^2}\right)^u = \frac{(\mu r/2)^{2u}}{4\pi^{3/2}r}\,\frac{\Gamma(\frac{1}{2}-u)}{\Gamma(1+u)}$$

$$= \int\frac{d^3\vec{q}}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}\exp\left[-u\,\log\left(\frac{q^2}{\mu^2}\right)\right] = \sum_n f_n(r,\mu)\,u^n$$



Asymptotically $\quad f_n(r,\mu) \sim \underbrace{\frac{1}{2\pi^2}}_{=\,-2\,\mathrm{Res}\left[F;\,u=\frac{1}{2}\right]}\mu\times 2^n$

Most dominant part is indep. of r !

9.4 ■ APPLICATIONS OF HIGH ORDER CALCULATIONS

The computation of radiative corrections entails evaluation of the amplitudes associated with many Feynman diagrams. Such computations are highly technical and require the use of computer programs which can perform symbolic manipulations and reductions to core integrals. By way of illustration, Figure 9.13 shows the result of an analytic calculation including terms of order α_s^4 of the static potential

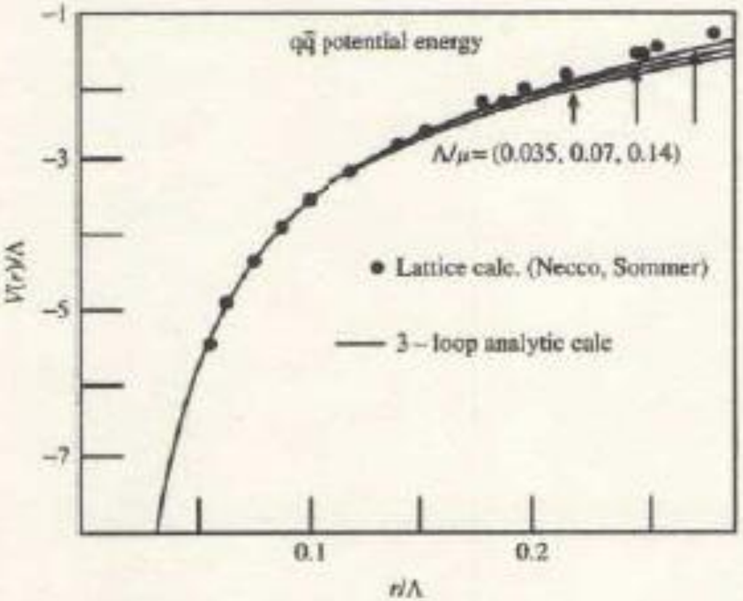
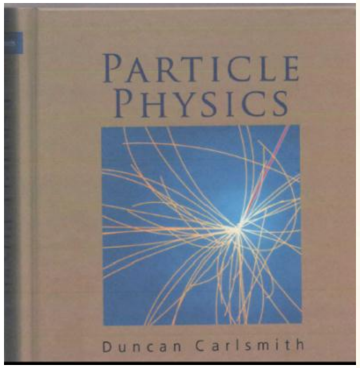


FIGURE 9.13 Computed static color interaction potential at three-loop order compared with the lattice computations. The curves correspond to several choices of renormalization scale. The points represent the results of independent lattice QCD calculations. [C. Anzai, Y. Kiyo, and Y. Sumino, Phys. Rev. Lett. **104**, 112003 (2010)]

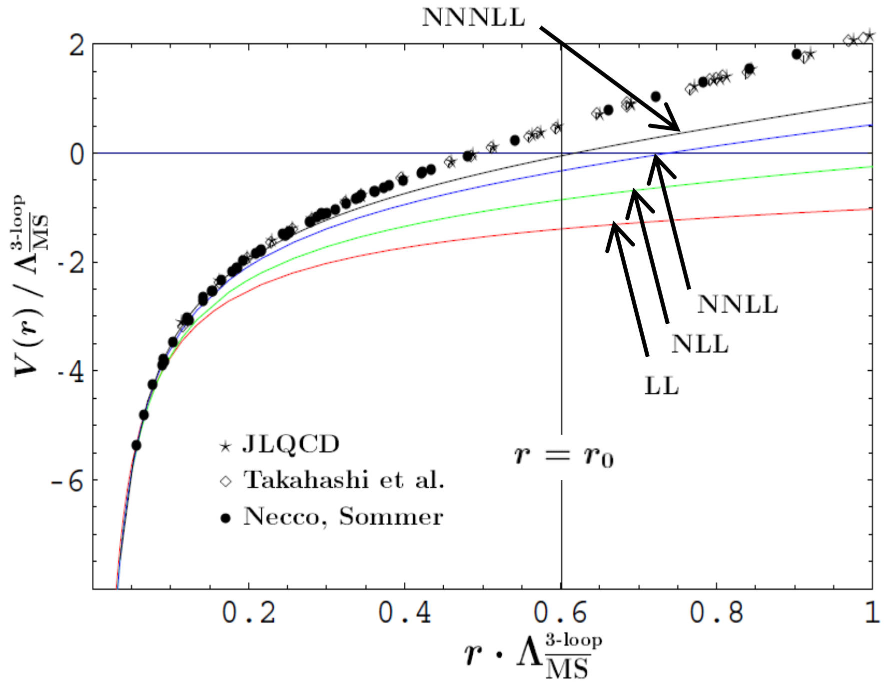


Interquark force

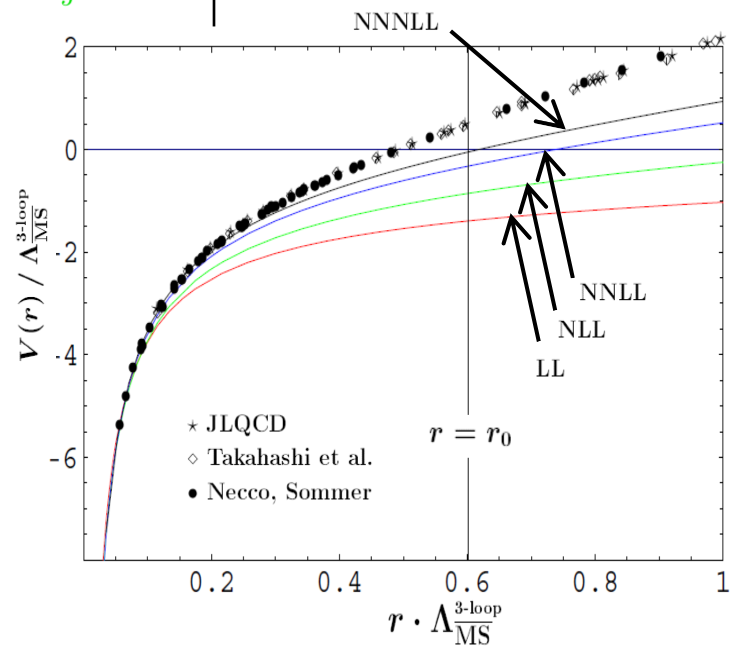
$$F(r) \equiv -\frac{d}{dr}V_{\rm QCD}(r)$$
$$\equiv -C_F\frac{\alpha_F(1/r)}{r^2}.$$

Renormalization-group equation: $\mu^2\frac{d}{d\mu^2}\alpha_F(\mu)=\beta_F(\alpha_F)$

$\Longrightarrow$ Due to the running of $\alpha_F(1/r)$, the attractive force $|F(r)|$ increases at large r.



Bottomonium spectrum up to $n = 3$ states



* Pert. QCD prediction including full $\mathcal{O}(\alpha_S^4 m)$ corrections to individual energy levels, as well as full $\mathcal{O}(\alpha_S^5 m)$ corrections to fine structure [$\alpha_S(M_Z) = 0.1181$]. Recksiegel, Y.S.

$$V_{\rm QCD}(r) = \underbrace{V_{\rm pert}(r;\mu_f)}_{\text{Wilson coeff.}} + \underbrace{\delta E(r;\mu_f)}_{\text{non-pert. contr.}}$$

$\delta E \to 0$ as $r \to 0$ $\sim \langle G^a_{\mu\nu}(0)^2\rangle\, r^3$

Comparison of lattice $V_{\rm QCD}(r)$ and $V_{\rm pert.}(r;\mu_f)$ at short distances

$$V_{\rm latt}(r) - V_{\rm pert}(r;\mu_f) = \delta E(r;\mu_f)$$

$\frac{1}{r\log r} \xleftrightarrow[\text{cancel}]{} \frac{1}{r\log r}$ $\qquad r^3$

Sensitive to relation between r_0 and $\Lambda_{\overline{MS}}$ or $\alpha_S(M_Z)$

$$r_0\Lambda^{\text{3-loop}}_{\overline{\rm MS}} = 0.574 \pm 0.042$$ Y.S.

c.f. Schrödinger functional method:

0.602 ± 0.048

Capitani, Lüscher, Sommer, Wittig

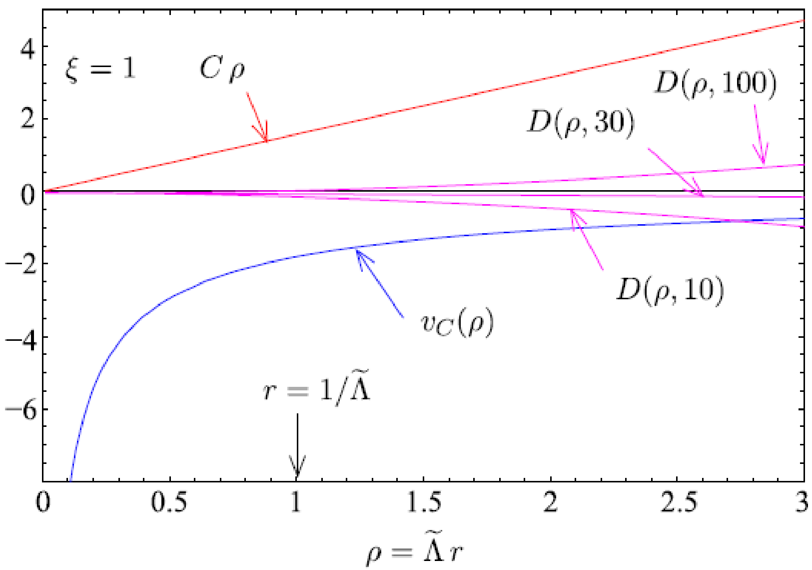
0.586 ± 0.048

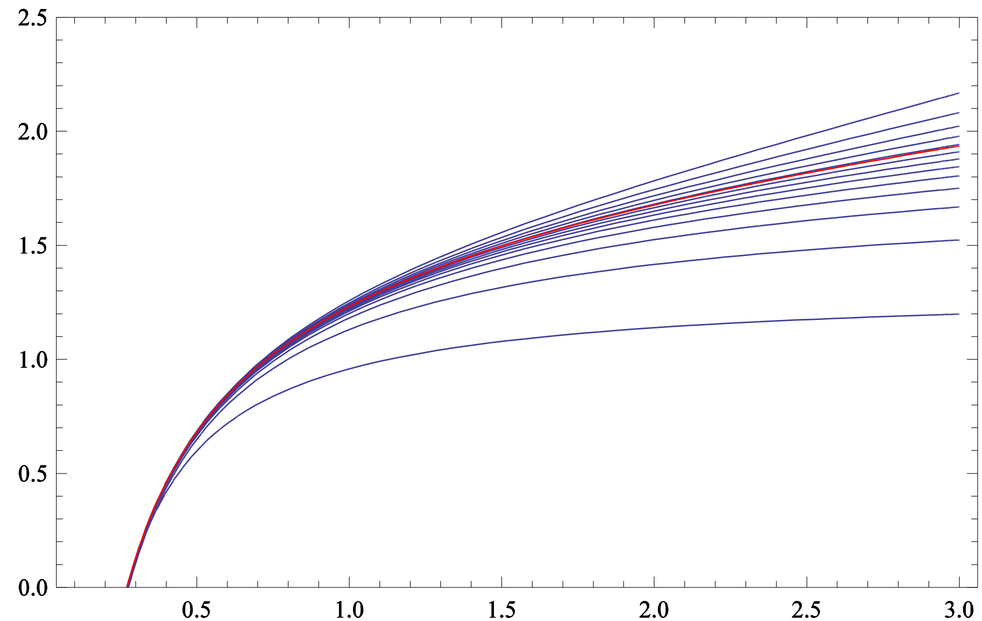
Necco, Sommer

Including 3-loop QCD pot.

$$r_0\Lambda^{3-\rm loop}_{\overline{\rm MS}} = 0.622^{+0.019}_{-0.015}$$

Brambilla,Tomo,Soto,Vairo



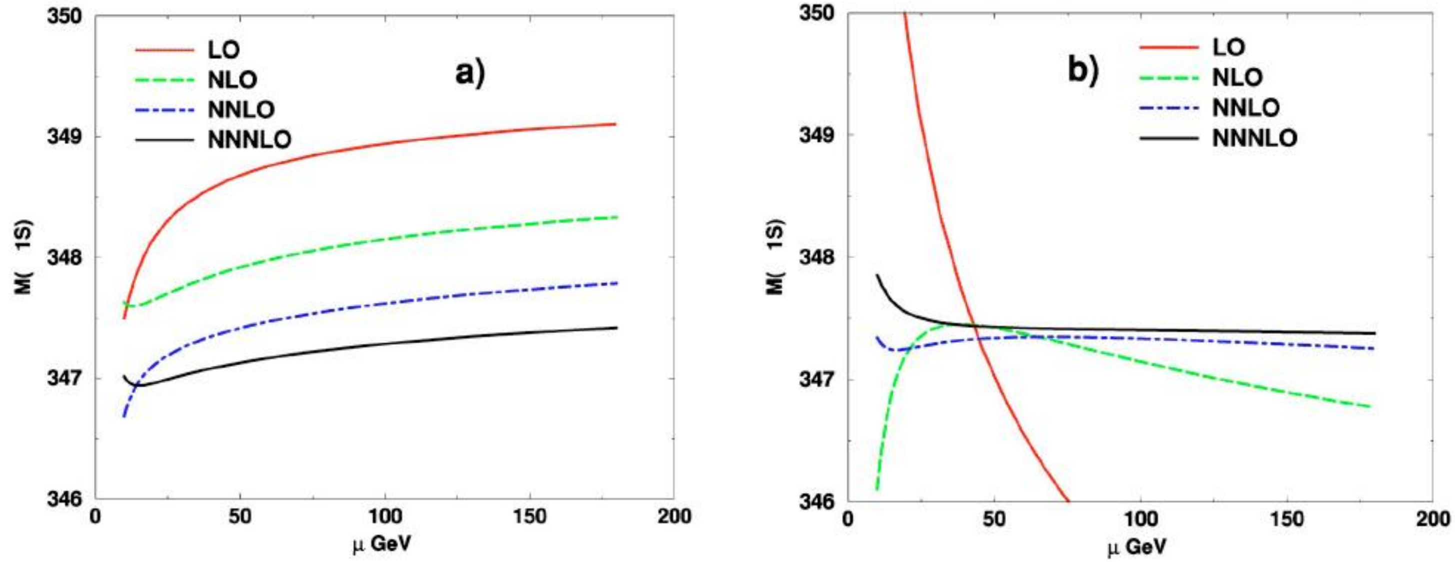


$$V_{\rm UV}(r;\mu_f) \equiv \int_{|q|>\mu_f} \frac{d^3q}{(2\pi)^3} \left[-4\pi C_F \frac{\alpha_V(q)}{q^2} \right]$$

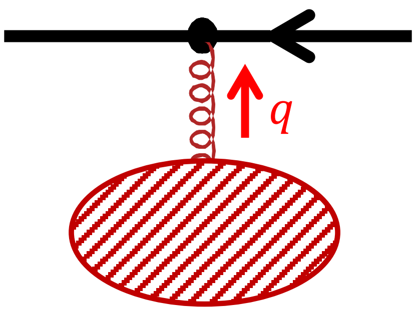
$$V_{\rm UV}(r;\mu_f) - \Big[V_C(r) + \sigma r \Big] = \text{const.} + \mathcal{O}(\mu_f^3 r^2)$$

for $r^{-1} \gg \mu_f \gg \Lambda.$

μ dependence and convergence of $M_{tt}(1S)$



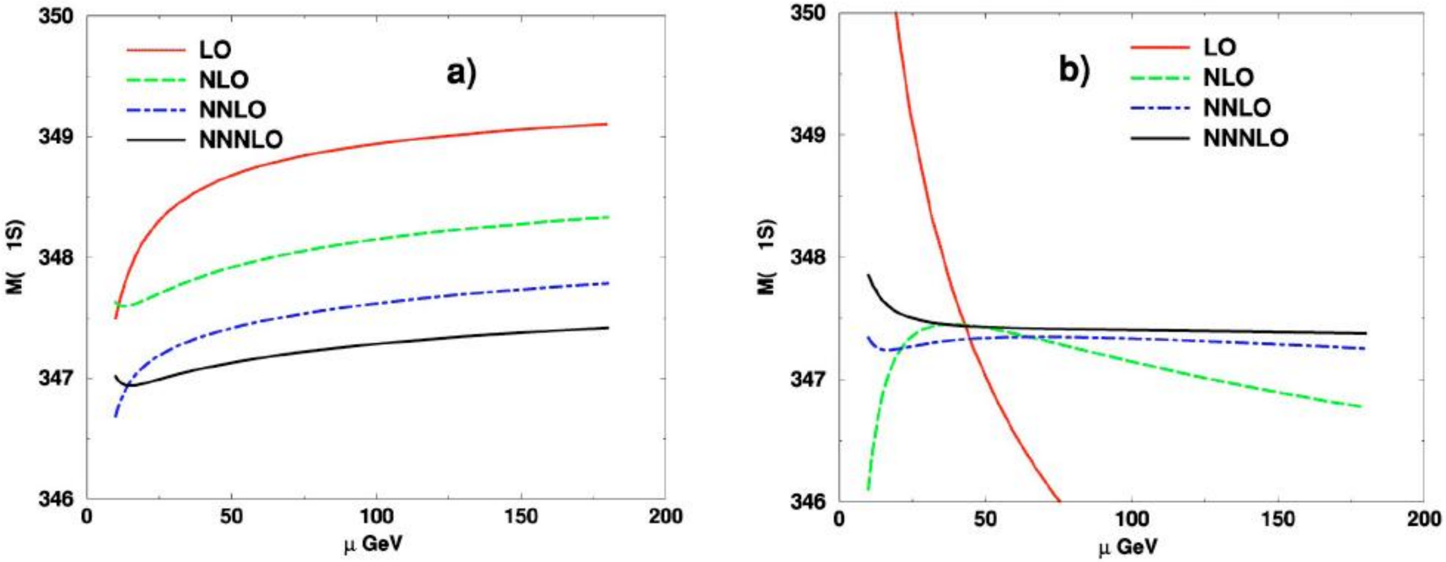
General feature of QCD beyond large β_0 or leading-log approx.



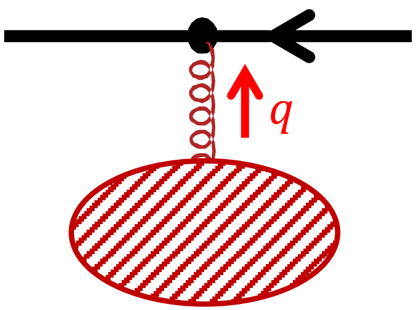
$\underline{A_\mu(q)}\, j^\mu(-q)$ $\qquad$ $j^\mu(x) = \delta^{\mu 0}\delta^3(\vec{x} - \vec{r}/2)$

Couples to total charge as $q \to 0$.

μ dependence and convergence of $M_{tt}(1S)$



General feature of QCD beyond large β_0 or leading-log approx.



$\underline{A_\mu(q)}\, j^\mu(-q)$ $\qquad$ $j^\mu(x) = \delta^{\mu 0}\delta^3(\vec{x} - \vec{r}/2)$

Couples to total charge as $q \to 0$.