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格子Yang-Mills理論の新しい定式化と非可換双対超伝導描像

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Talk based on: **Phys. Rev. D 87, 054011 (2013) , arXiv:1212.6512**

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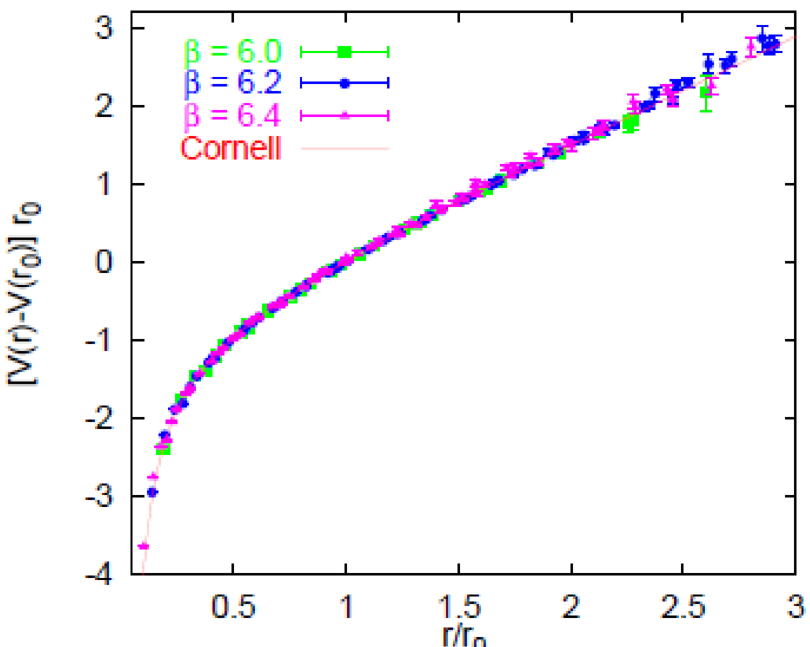
Introduction

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]

Non-Abelian Wilson loop $\left\langle \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]\right\rangle_{\mathrm{YM}}^{\text{no GF}} \sim e^{-\sigma_{NA}|S|}$



$$V(r) = -C\frac{g_{\mathrm{YM}}^2(r)}{r} + \sigma r$$

$$F(r) = -\frac{d}{dr}V(r) = -C\frac{g_{\mathrm{YM}}^2(r)}{r^2} - \sigma + \cdots \quad (C, \sigma > 0)$$

G.S. Bali, [hep-ph/0001312], Phys. Rept. **343, 1–136 (2001)**

dual superconductivity

- Dual superconductivity is a promising mechanism for quark confinement. [Y.Nambu (1974). G.’t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]

superconductor

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

dual superconductor

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks

The evidence for dual superconductivity

To establish the dual superconductivity picture, we must show that **the magnetic monopole plays a dominant role for quark confinement:**

Many preceding studies based on the Abelian projection: $U_{x,\mu} = X_{x,\mu} V_{x,\mu}$

The gauge link is decomposed into the Abelian (diagonal) part V and the remainder (off-diagonal) part X

$$U_{x,\mu} = u^0_{x,\mu}\mathbf{1} + \sum_{j=1}^{3} u^j_{x,\mu}\sigma^j, \text{ with } (u^0_{x,\mu})^2 + \sum_{j=1}^{3}\left(u^j_{x,\mu}\right)^2 = 1$$

$$V_{x,\mu} = \frac{u^0_{x,\mu}\mathbf{1} + u^3_{x,\mu}\sigma^3}{\sqrt{(u^0_{x,\mu})^2 + (u^3_{x,\mu})^2}}, \quad X_{x,\mu} = U_{x,\mu} V^{\dagger}_{x,\mu}$$

SU(2) case

Abelian-projected Wilson loop $\left\langle \exp\left\{ ig \oint_C dx^\mu A^3_\mu(x) \right\} \right\rangle^{\text{MAG}}_{\text{YM}} \sim e^{-\sigma_{Abel}|S|}$!?

The evidence for dual superconductivity(cont')

- ☐ Abelian dominance in the string tension [Suzuki & Yotsuyanagi, 1990]
- ☐ Abelian magnetic monopole dominance in the string tension [Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]
- ☐ Measurement of (Abelian) dual Meissner effect
- ◆ Observation of chromo-electric flux tubes and Magnetic current due to chromo-electric flux
- ◆ Type the super conductor is the order between Type I and Type II [Y.Matsubara, et.al. 1994]

Problems:

- ✓ These are only obtained in the case of special gauge such **as maximal Abelian gauge (MAG),**
- ✓ gauge fixing breaks the gauge symmetry as well as color symmetry (global symmetry).

A new lattice formulation

- ***We have presented a new lattice formulation of Yang-Mills theory, that can establish "Abelian" dominance and magnetic monopole dominance in the gauge independent way (gauge-invariant way)***

We have proposed the decomposition of gauge link,

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

which can extract the relevant mode *V* for quark confinement.

- For SU(2) case, the decomposition is a lattice compact representation of the *Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.*
- For SU(N) case, the formulation is the extension of the SU(2) case.

The path integral formulation by Kondo-Murakami-Shinohara;

SU(2) case: Eur. Phys. J. C 42, 475 (2005), Prog. Theor. Phys. 115, 201 (2006).

SU(N) case: Prog.Theor. Phys. 120, 1 (2008)

■ SU(2) Yang-Mills Theory

- We have presented the compact representation of Cho-Duan-Ge-Faddeev-Niemi (CDGFN) decomposition for SU(2) case on a lattice, i.e., **the decomposition of gauge link, *U=XV.***

quark-antiquark potential from Wilson loop operator shows

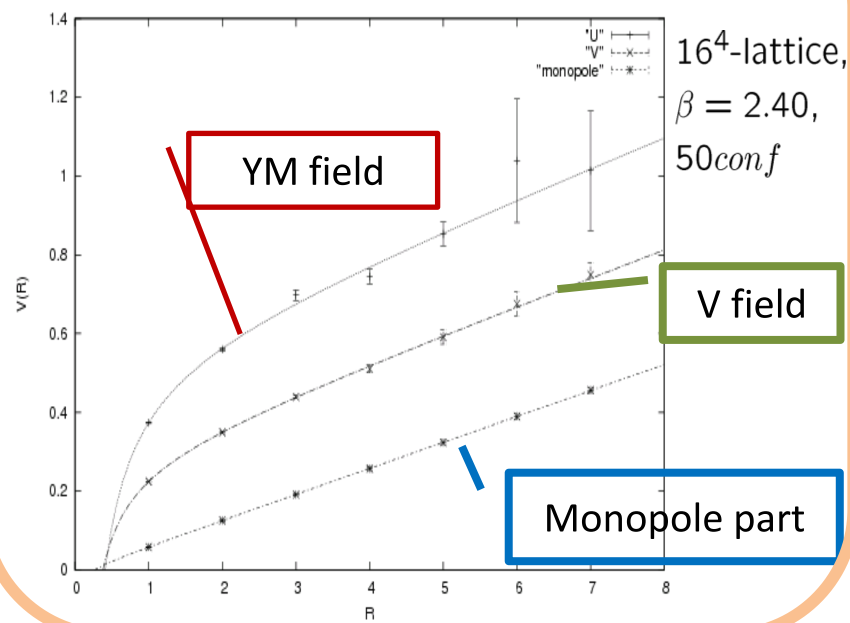
- ***gauge-independent "Abelian" dominance*** : the decomposed V field reproduced the potential of original YM field.

  $$\sigma_{full} \sim \sigma_V \quad (93 \pm 16\%)$$

- ***gauge-independent monopole dominance*** :the string tension is almost reproduced by only magnetic monopole part.

  $$\sigma_{full} \sim \sigma_{monopole} \quad (88 \pm 13\%)$$

$$V(R) = c + \frac{\alpha}{R} + \sigma R$$



arXiv:0911.0755 [hep-lat], Phys.Lett. B645 67-74 (2007)

A new formulation of lattice SU(3) Yang-Mills theory

A new formulation of Yang-Mills theory (on a lattice)

Decomposition of SU(N) gauge links

- The decomposition as the extension of the SU(2) case.
- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

□ SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$

□ SU(3) Yang-Mills link variables: Two options

maximal option : $U(1) \times U(1) \subset SU(3)$

✓ Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓ Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

minimal option::Wilson loop for the fundamental representation

- **Two reformulations** written in terms of different variables **are equivalent to each other.** This is simply the choice of the coordinates in the space of gauge field configurations.

- **The difference between two options, i.e, maximal or minimal, arises when we choose an operator to be calculated.**
  - Wilson loop operator is uniquely defined by giving a representation, to which the source quark belongs.
  - **the Wilson loop operator in the fundamental representation leads us to the minimal option**
  - which is shown in the process of deriving **a non-Abelian Stokes theorem** for the Wilson loop operator by Kondo **PRD77 085929(2008)**

The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

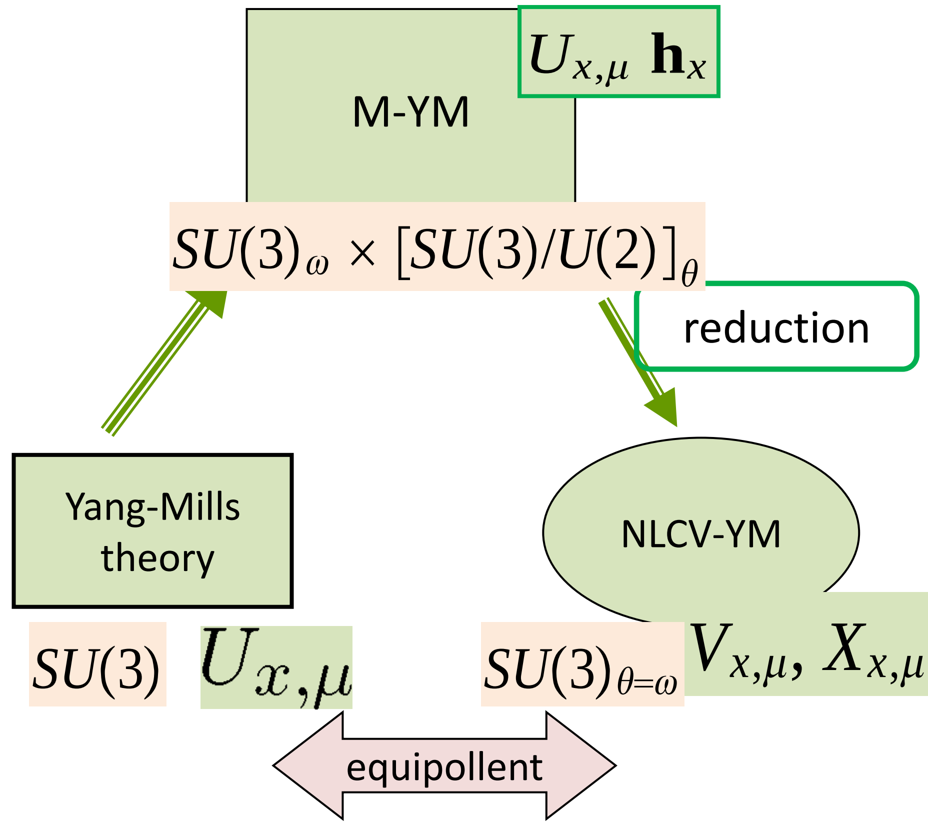
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_{x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Defining equation for the decomposition

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}^\dagger_{x,\mu}(\det \hat{L}_{x,\mu})^{1/N} g_x^{-1} \quad V_{x,\mu} = X^\dagger_{x,\mu}U_x, = g_x\hat{L}_{x,\mu}U_x,(\det \hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L^\dagger_{x,\mu}}\right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu}$$

continuum version by continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

The defining equation and implication to the Wilson loop for the fundamental representation

K.-I. Kondo, Phys.Rev.D77:085029,2008

K.-I. Kondo, A. Shibata arXiv:0801.4203 [hep-th]

By inserting the complete set of the coherent state $|\xi_x, \Lambda\rangle$ at every site on the Wislon loop C, $1 = \int |\xi_x, \Lambda\rangle d\mu(\xi_x) \langle \Lambda, \xi_x|$ we obtain

$$W_C[U] = \mathrm{tr}\left(\prod_{<x>\in C} U_{x,\mu} \right) = \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x) \langle \Lambda, \xi_x | U_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle$$

$$= \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x) \langle \Lambda, |(\xi_x^\dagger X_{x,\mu} \xi_x)(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu})|, \Lambda \rangle$$

where we have used $\xi_x \xi_x^\dagger = 1$.

For the stability group of $\tilde{H}$, the 1st defining equation

$$\xi V_{x,\mu} \xi^\dagger \in \tilde{H} \iff [\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}, \tilde{H}] \iff \mathbf{h}_x V_{x,\mu} - V_{x,\mu} \mathbf{h}_{x+\mu} = 0$$

implies that $|\Lambda\rangle$ is eigenstate of $\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}$:

$$(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu})|\Lambda\rangle = |\Lambda\rangle e^{i\phi}, \quad e^{i\phi} := \langle \Lambda | \xi_x^\dagger V_{x,\mu} \xi_{x+\mu} | \Lambda \rangle = \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle.$$

Then we have

$$W_C[U] = \int d\mu(\xi_x) \rho[X;\xi] \prod_{<x,x+\mu>\in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle$$

$$\rho[X;\xi] := \prod_{<x>\in C} \langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle$$

Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ describing the original Yang-Mills theory and color fields.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory
- The configuration of the color fields $\mathbf{h}_x$ can be determined by the reduction condition such that the reduction functional is minimized for given $U_{x,\mu}$

$$F_{\rm red}[\mathbf{h}_x; U_{x,\mu}] = \sum_{x,\mu} \mathrm{tr}\left\{ (D_\mu^\epsilon[U]\mathbf{h}_x)^\dagger (D_\mu^\epsilon[U]\mathbf{h}_x) \right\}$$

$$SU(3)_\omega \times [SU(3)/U(2)]_\theta \to SU(3)_{\omega=\theta}$$

- This is invariant under the gauge transformation $\theta=\omega$
- The extended gauge symmetry is reduced to the same symmetry as the original YM theory.
- We choose a reduction condition of the same type as the SU(2) case

Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection

$$W_C[\mathcal{A}] = \int [d\mu(\xi)]_\Sigma \exp\left(-ig \int_{S:C=\partial\Sigma} dS^{\mu\nu} \sqrt{\frac{N-1}{2N}} \mathrm{tr}(2\mathbf{h}(x)\mathcal{F}_{\mu\nu}[\mathcal{V}](x))\right)$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left(ig\sqrt{\frac{N-1}{2N}}(k, \Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j, N_\Sigma)\right)$$

magnetic current $k := \delta^* F = {}^* dF$, $\Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1}$

electric current $j := \delta F$, $N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$

$\Delta = d\delta + \delta d$, $\Theta_\Sigma := \int_\Sigma d^2 S^{\mu\nu}(\sigma(x))\delta^D(x - x(\sigma))$

k and j are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

K.-I. Kondo PRD77 085929(2008)

The lattice version is defined by using plaquette:

$$\Theta^8_{\mu\nu} := -\arg \mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x\right) V_{x,\mu} V_{x+\mu,\mu} V^\dagger_{x+\nu,\mu} V^\dagger_{x,\nu}\right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu \Theta^8_{\alpha\beta},$$

Test of dual super conductivity on a lattice

- Linear potential:
  - Restricted field V dominance (so called "Abelian" dominance)
  - Non-Abelian magnetic monopole dominance

- Chromomagnetic flux: Measurement of the chromo-magnetic field
  - chromo-electric flux tube from quark and antiquark source
  - Magnetic (monopole) current due to magnetic monopole condensation

■ SU(3) Yang-Mills theory

- In confinement of fundamental quarks, a restricted non-Abelian variable $\boldsymbol{V}$, and the extracted non-Abelian magnetic monopoles play the dominant role (dominance in the string tension), in marked contrast to the Abelian projection.

gauge independent "Abelian" dominance

$$\frac{\sigma_V}{\sigma_U} = 0.92$$

$$\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82$$

Gauge independent non-Abalian monople dominance

$$\frac{\sigma_M}{\sigma_U} = 0.85$$

$$\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76$$

U^* is from the table in R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. B517, 377 (1998).
(based on Abelian projection)

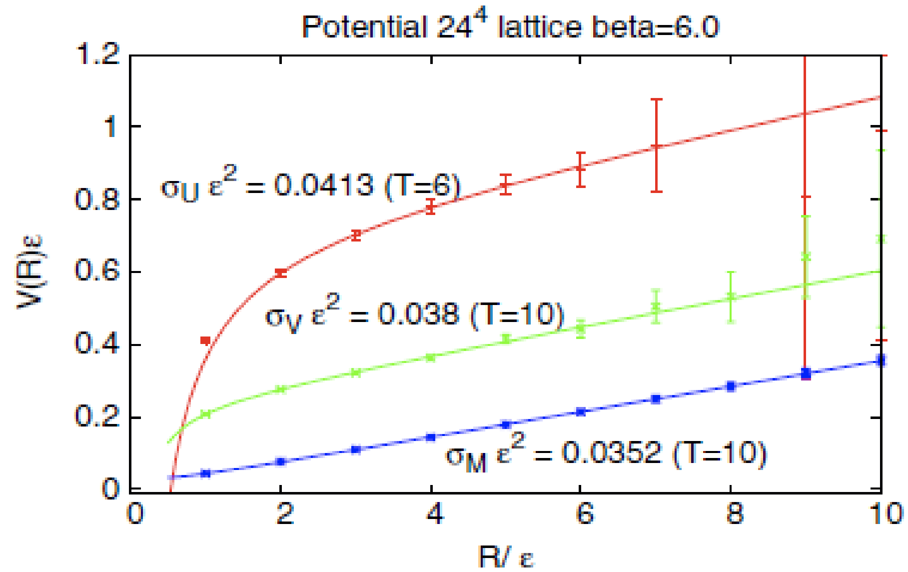


FIG. 1 (color online). $SU(3)$ quark-antiquark potentials as functions of the quark-antiquark distance R: (from tob to bottom) (i) full potential $V_f(R)$ (red curve), (ii) restricted part $V_r(R)$ (green curve), and (iii) ma;gnetic-monopole part $V_m(R)$ (blue curve), measured at $\beta = 6.0$ on 24^4 using 500 configurations where ϵ is the lattice spacing.

PRD 83, 114016 (2011)

Chromo-electric flux

$$\rho_W = \frac{\langle \mathrm{tr}(WLU_pL^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

By Adriano Di Giacomo et.al.

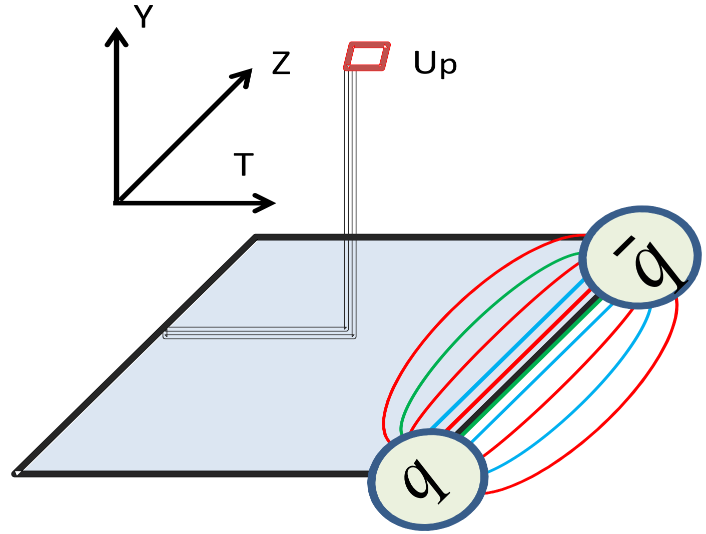
[Phys.Lett.B236:199,1990] [Nucl.Phys.B347:441-460,1990]

Gauge invariant correlation function: This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3)

$$\rho_W \overset{\epsilon\to 0}{\simeq} \frac{\mathrm{tr}(ig\epsilon\mathcal{F}_{\mu\nu}LWL^\dagger)}{\mathrm{tr}(LWL^\dagger)} =: \langle g\epsilon\mathcal{F}_{\mu\nu}\rangle_{q\bar{q}}$$

$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\,\rho_W(x)$$

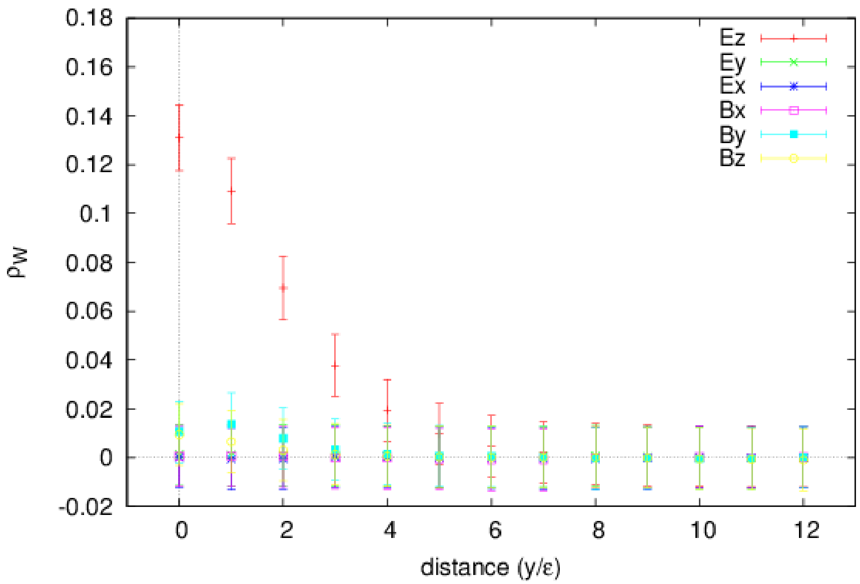
$$\mathrm{tr}(U_pLWL^\dagger)$$



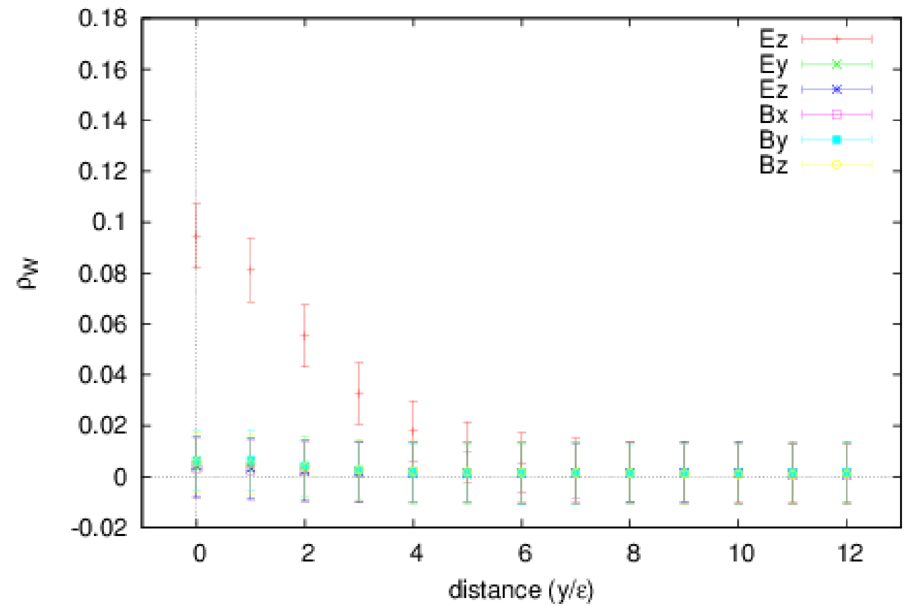
Chromo-electric flux

- YM gauge configurations: by standard Wilson action on a 24^4 lattice with β=6.2.
- The gauge link decomposition: the color field configuration is obtained by solving the reduction condition of minimizing the functional , and the decomposition is obtained by using the formula of the decompoition.
- measurement of the Wilson loop: APE smearing technique to reduce noises.
- measure correlation of the restricted U(2) field, as well as the original YM field.

Original YM filed

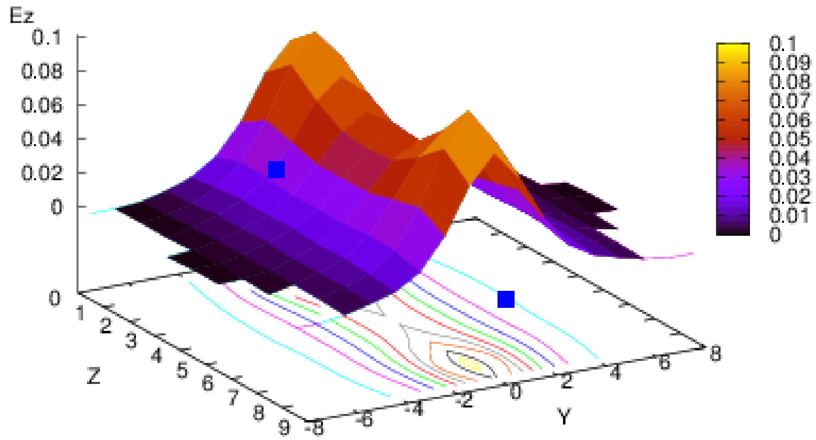


∝ Restricted U(2) field

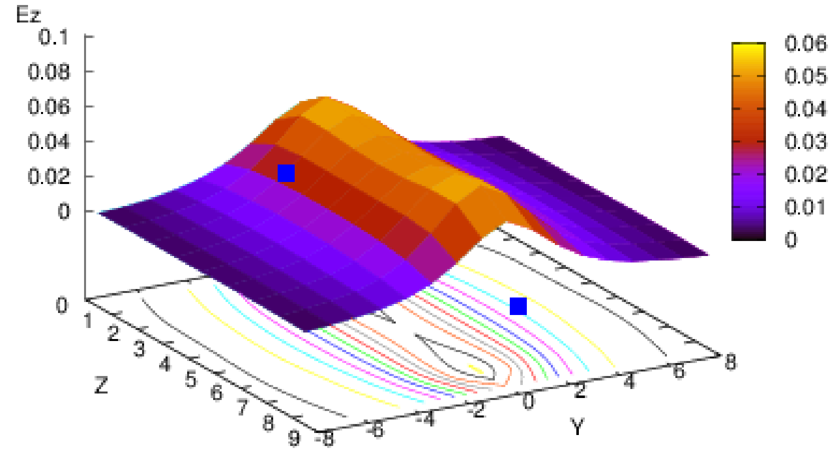


Chromo-electric (color flux) Flux Tube

Original YM filed



Restricted U(2) field



A pair of quark-antiquark is placed on z axis as the 9x9 Wilson loop in Z-T plane. Distribution of the chromo-electronic flux field created by a pair of quark-antiquark is measured in the Y-Z plane, and the magnitude is plotted both 3-dimensional and the contour in the Y-Z plane.

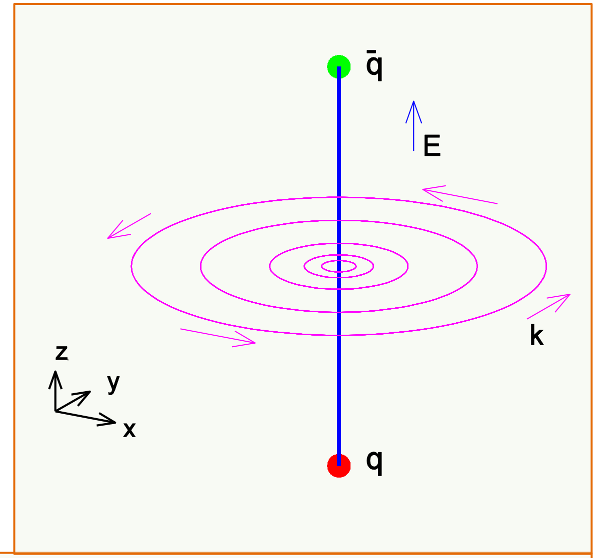
Flux tube is observed for the restricted U(2) field case.

Magnetic current induced by quark and antiquark pair

Yang-Mills equation (Maxwell equation) for $\mathbf{V}_\mu$ field,
the magnetic monopole (current) can be calculated as

$$\mathbf{k} = {}^*dF[\mathbf{V}] \,,$$

$F[\mathbf{V}]$ is the field strength 2-form of V_μ field
d the exterior derivative and $*$ denotes the Hodge dual.



$\mathbf{k} \neq \mathbf{0} \Rightarrow$

signal of the monopole condensation

the field strength is given by $F[\mathbf{V}] = d\mathbf{V}$

the Bianchi identity : $\mathbf{k} = {}^*d^2\mathbf{V} = 0$

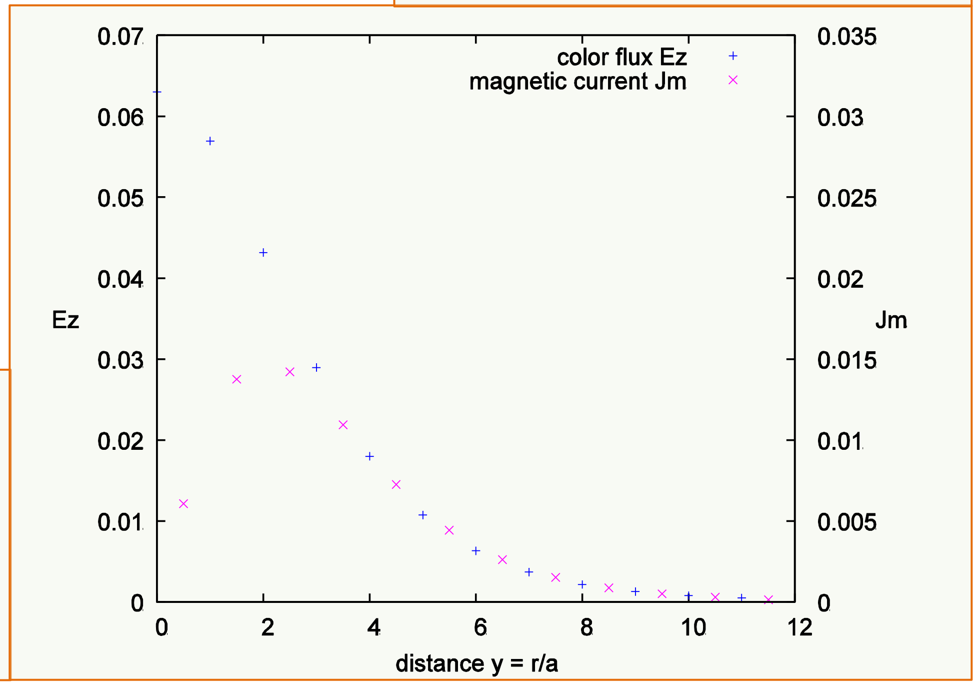


Figure: (upper) positional relationship of chromo-electric flux and magnetic current. (lower) combination plot of chromo-electric flux (left scale) and magnetic current(right scale).

Type of Yang-Mills vacuum

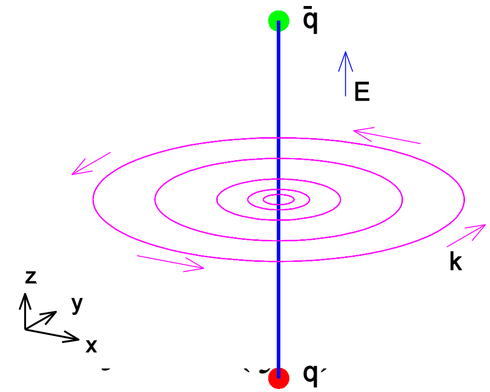
Type of dual superconductivity (Ginzburg-Landau theory)

Ginzburg–Landau equation

$$D_\mu D^\mu \phi - \lambda(\phi^* \phi - \mu^2/\lambda^2)\phi = 0$$

Ampere equation

$$\partial^\nu F_{\mu\nu} + iq[\phi^*(D_\mu \phi) - (D_\mu \phi)^* \phi] = 0$$



J.R.Clem J. low Temp. Phys. 18 427 (1975)

$$\phi[y] = \frac{\Phi_0}{2\pi} \frac{1}{\sqrt{2}\,\lambda} \frac{y}{\sqrt{y^2 + \xi^2}}$$

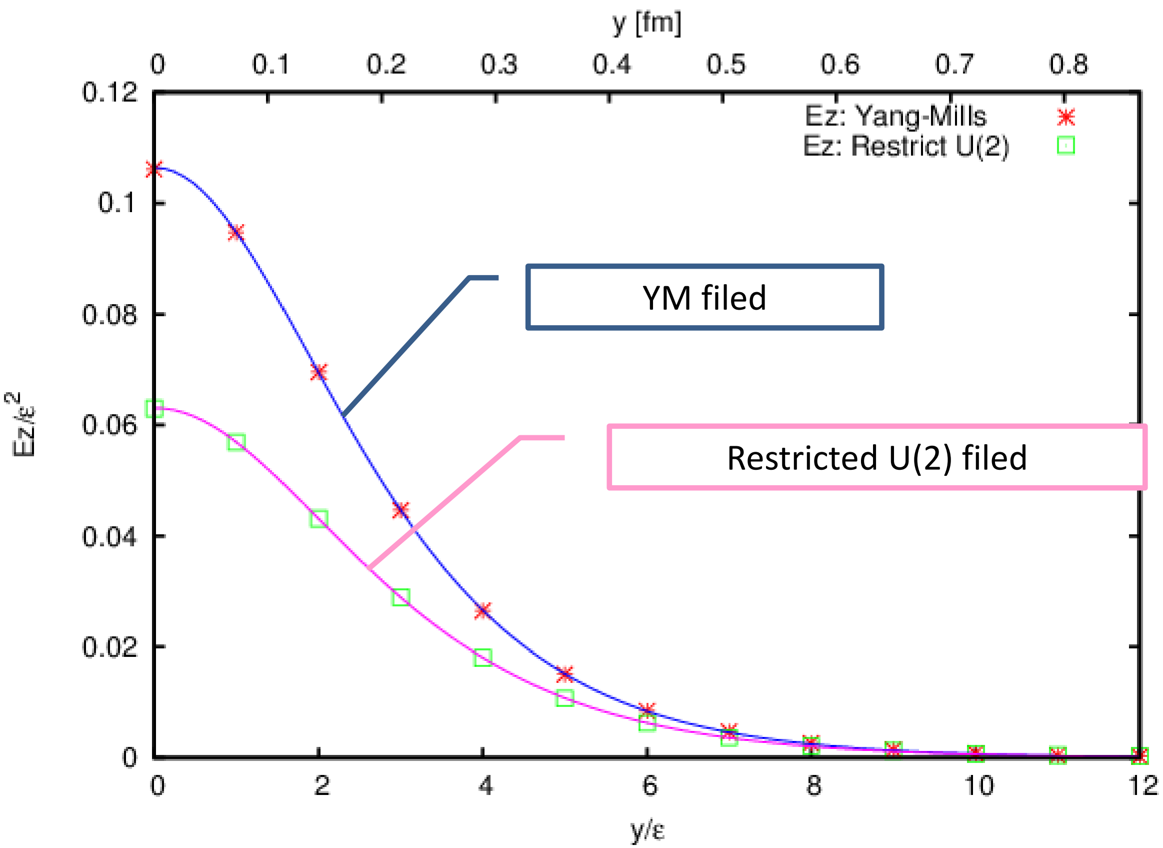
The profile of chromo-electric flux in the super conductor is given by

$$E_z[y] = \frac{\Phi_0}{2\pi} \frac{1}{\xi\lambda} \frac{K_0(R/\lambda)}{K_1(\xi/\lambda)}, \quad R = \sqrt{y^2 + \xi^2}$$

K_ν : the modified Bessel function of the ν-th order, λ the parameter corresponding to the London penetration length, ξ a variational core radius parameter, and Φ_0 external flux.

❖ this formula is for the super conductor of U(1) gauge field.

Type of dual superconductivity (Ginzburg-Landau parameter)



fitting by $E_z(y) = aK_0(\sqrt{b^2y^2 + c^2})$

with $a = \phi/[2\pi\xi/\lambda K_1(\xi/\lambda)]$,

$b = 1/\lambda$, $c = \xi/\lambda$

Ginzburg-Landau (GL) parameter

$$\kappa = \sqrt{2}/(\xi/\lambda)\sqrt{1 - K_0^2(\xi/\lambda)/K_1^2(\xi/\lambda)}.$$

Type I $\kappa < \kappa_c = 1/\sqrt{2} \simeq 0.707$

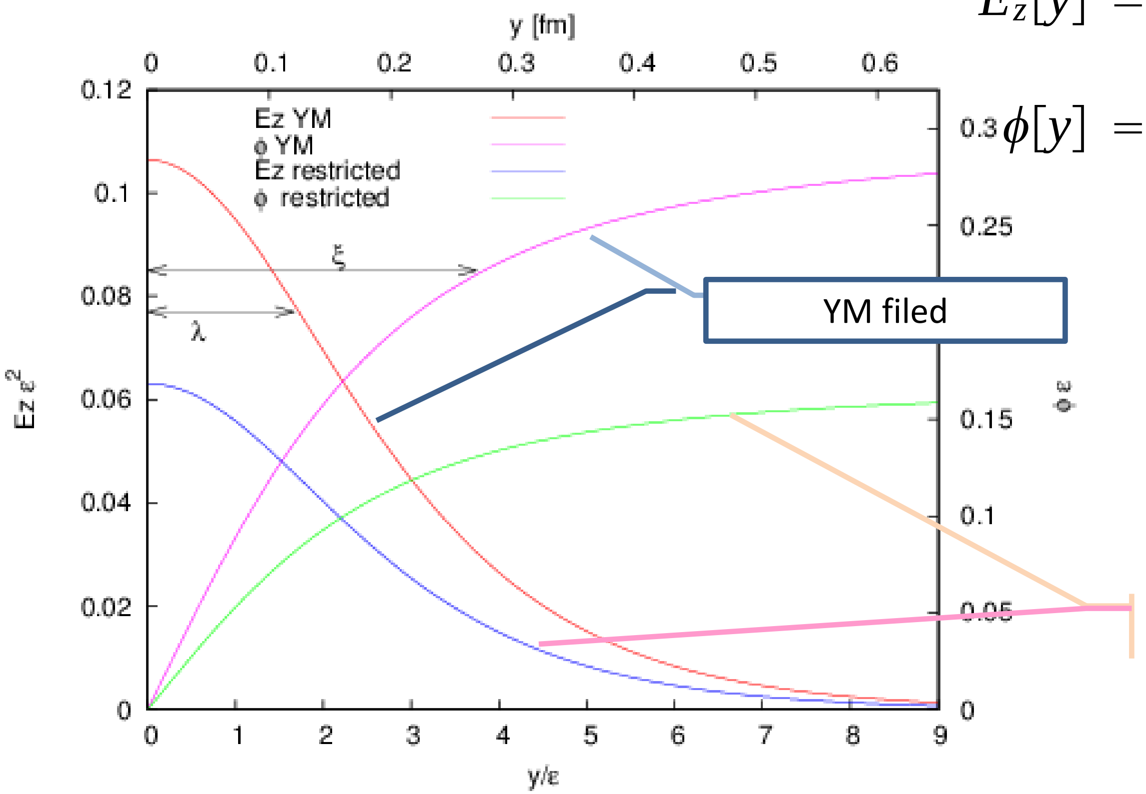
Type || $\kappa > \kappa_c$

	$a\epsilon^2$	$b\epsilon$	c	λ/ϵ	ζ/ϵ	ξ/ϵ	Φ	κ
SU(3) YM field	0.804 ± 0.04	0.598 ± 0.005	1.878 ± 0.04	1.672 ± 0.014	3.14 ± 0.09	3.75 ± 0.12	4.36 ± 0.3	0.45 ± 0.01
restricted field	0.435 ± 0.03	0.547 ± 0.007	1.787 ± 0.05	1.828 ± 0.023	3.26 ± 0.13	3.84 ± 0.19	2.96 ± 0.3	0.48 ± 0.02

Type of dual superconductivity: fitted solutions

$$E_z[y] = \frac{\Phi_0}{2\pi}\frac{1}{\xi\lambda}\frac{K_0(R/\lambda)}{K_1(\xi/\lambda)}, R = \sqrt{y^2+\xi^2}$$

$$\phi[y] = \frac{\Phi_0}{2\pi}\frac{1}{\sqrt{2}\,\lambda}\frac{y}{\sqrt{y^2+\xi^2}}$$



	$a\epsilon^2$	$b\epsilon$	c	λ/ϵ	ζ/ϵ	ξ/ϵ	Φ	κ
SU(3) YM field	0.804 ± 0.04	0.598 ± 0.005	1.878 ± 0.04	1.672 ± 0.014	3.14 ± 0.09	3.75 ± 0.12	4.36 ± 0.3	0.45 ± 0.01
restricted field	0.435 ± 0.03	0.547 ± 0.007	1.787 ± 0.05	1.828 ± 0.023	3.26 ± 0.13	3.84 ± 0.19	2.96 ± 0.3	0.48 ± 0.02

type of the dual superconductivity(summary)

- YM field type I :

$$\kappa = 0.45 \pm 0.01. \ \lambda = 0.1207(17)\text{fm} \ \ \xi = 0.2707(86)\text{fm}$$

consistent with Cea, Cosmai and Papa, PRD86(054501) (2012)

- restricted U(2) field (minimal option) type I :

$$\kappa = 0.48 \pm 0.02, \quad \lambda = 0.132(3)\text{fm} \ \ \xi = 0.277(14)\text{fm}.$$

- comparison with other results:
  - MA gauge Abelian Projection : border of type I and type II $\kappa = 0.5 - 1$

    Yoshimi Matsubara, Shinji Ejiri and Tsuneo Suzuki, NPB Poc. suppl 34, 176 (1994)
  - YM field: type II $\kappa = 1.2 - 1.3$

    N. Cardoso, M. Cardoso, P. Bicudo, arXiv:1004.0166
- SU(2) case

■ GL parameter and type of dual superconductor SU(2)

Talk by S.Kato, 27aRE8

$$\kappa_U = 0.717 \pm 0.208 \qquad \kappa_V = 0.491 \pm 0.150$$

This result shows the dual superconductor for the SU(2) lattice Yang-Mills theory is the border between type I and type II.

Penetration depth λ and coherence lemgth ξ is obtained as (ε(β=2.5)=0.0832fm),

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- Cea,Cosmai,Papa(2012); YM, β=2.52,2.55,2.6, $L^4=20^4$.

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- Bali,Schlichter,Schilling(2009); β=2.5115, $L^4=32^4$

  $$\kappa = 0.594(\lambda = 1.84^{+20}_{-24}, \xi = 3.10^{+43}_{-35}) \rightarrow \lambda = 0.153 fm, \xi = 0.258 fm$$

Summary

- We investigate our proposal: non-Abelain dual superconductivity picture for SU(3) Yang-Mills theory as the mechanism of quark confinement.
- Applying a new formulation of Yang-Mills theory, we study non-Abelian dual Meissner effect.
- Extracting the dominant mode by using the decomposition of link variables: ***U=XV*** : decomposition based on the stability group $U(2)$
- restricted $U(2)$ field (V-field) dominance in string tension
- non-Abelian magnetic monopole dominance in string tension
- Observation of chromo-electric flux tube and non-Abelian magnetic current (monopole) induced from quark-antiquark pair
- Determination of type of the dual superconductivity : rather type I

outlook

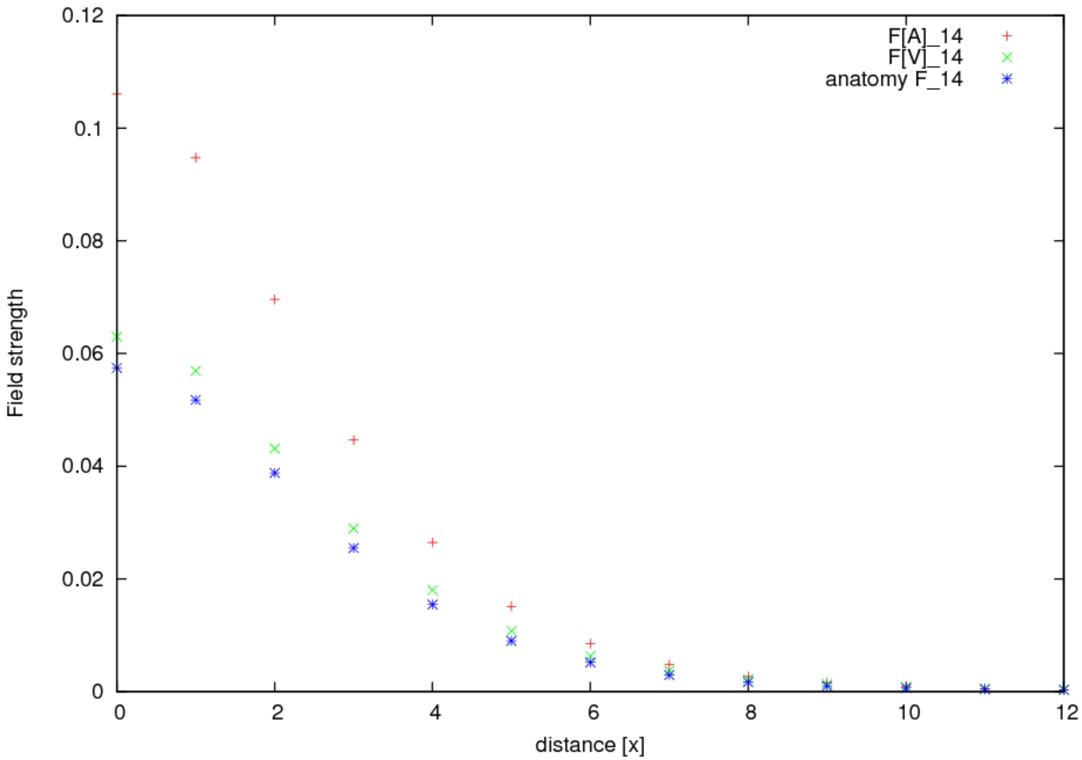
- Interaction among chromo-electric flux tubes:
  - Attractive (type I) of repulsive (type II) ?
  - Reflecting internal non-Abelian character?

- Confinement and deconfienment phase transition in the finite temperature
  - Phase transition and magnetic monopole condensation
  - Phase transition of dual super condacter in finite temperature

Thank you for your attention.

appendix

Measurement by three types of operators



Comparison of the correlation for the different Wilson line operator.

$F[A]_{14}$: Wilson line by using the original YM field (U).

$F[V]_{14}$: Wilson line by using the decomposed restricted U(2) field (V).

Anatomy F_{14}: Wilson line by using the original YM field as the quark source, and the restricted U(2) field (V) as the probed part ($\boldsymbol{L V_p L^{+}}$) .

The defining equation and implication to the Wilson loop for the fundamental representation

K.-I. Kondo, Phys.Rev.D77:085029,2008

K.-I. Kondo, A. Shibata arXiv:0801.4203 [hep-th]

By inserting the complete set of the coherent state $|\xi_x, \Lambda\rangle$ at every site on the Wislon loop C, $1 = \int |\xi_x, \Lambda\rangle d\mu(\xi_x)\langle\Lambda, \xi_x|$ we obtain

$$W_C[U] = \mathrm{tr}\left(\prod_{<x>\in C} U_{x,\mu}\right) = \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x)\langle\Lambda, \xi_x|U_{x,\mu}|\xi_{x+\mu}, \Lambda\rangle$$

$$= \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x)\langle\Lambda, |(\xi_x^\dagger X_{x,\mu}\xi_x)(\xi_x^\dagger V_{x,\mu}\xi_{x+\mu})|, \Lambda\rangle$$

where we have used $\xi_x\xi_x^\dagger = 1$.

For the stability group of $\tilde{H}$, the 1st defining equation

$$\xi V_{x,\mu}\xi^\dagger \in \tilde{H} \iff [\xi_x^\dagger V_{x,\mu}\xi_{x+\mu}, \tilde{H}] \iff \mathbf{h}_x V_{x,\mu} - V_{x,\mu}\mathbf{h}_{x+\mu} = 0$$

implies that $|\Lambda\rangle$ is eigenstate of $\xi_x^\dagger V_{x,\mu}\xi_{x+\mu}$:

$$(\xi_x^\dagger V_{x,\mu}\xi_{x+\mu})|\Lambda\rangle = |\Lambda\rangle e^{i\phi}, \quad e^{i\phi} := \langle\Lambda|\xi_x^\dagger V_{x,\mu}\xi_{x+\mu}|\Lambda\rangle = \langle\Lambda, \xi_x|V_{x,\mu}|\xi_{x+\mu}, \Lambda\rangle.$$

Then we have

$$W_C[U] = \int d\mu(\xi_x)\rho[X;\xi] \prod_{<x,x+\mu>\in C} \langle\Lambda, \xi_x|V_{x,\mu}|\xi_{x+\mu}, \Lambda\rangle$$

$$\rho[X;\xi] := \prod_{<x>\in C} \langle\Lambda, \xi_x|X_{x,\mu}|\xi_{x+\mu}, \Lambda\rangle$$

The defining equation and the Wilson loop for the fundamental representation (2)

By using the expansion of $X_{x,\mu}$: the 2nd defining equaiton, $\mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = \mathbf{0}$, derives

$$\langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = \mathrm{tr}(X_{x,\mu})/\mathrm{tr}(\mathbf{1}) + 2\mathrm{tr}(X_{x,\mu}\mathbf{h}_x)$$
$$= 1 + 2ig\epsilon tr(\mathcal{X}_\mu(x)\mathbf{h}(x)) + O(\epsilon^2).$$

Then we have $\rho[X;\xi] = 1 + O(\epsilon^2)$.

Therefore, we obtain

$$W_c[U] = \int d\mu(\xi_x) \prod_{<x,x+\mu>\in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = W_C[V]$$

By using the non−Abalian Stokes theorem, Wilson loop along the path C is written to area integral on Σ :$C = \partial\Sigma$;

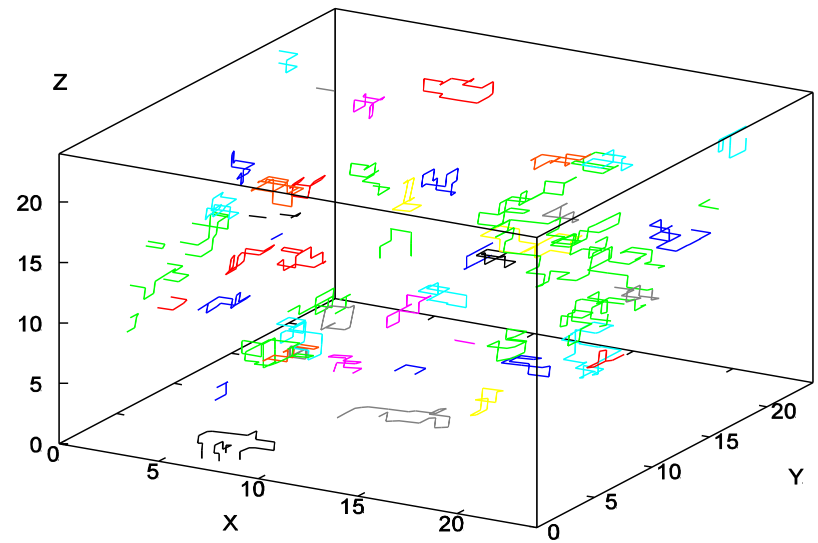
$$W_C[\mathcal{A}] := \mathrm{tr}\left[P\exp\left(-ig \oint_C dx^\mu \mathcal{A}_\mu(x) \right) \right] / \mathrm{tr}(\mathbf{1}) = \int d\mu_\Sigma(\xi) \exp\left(\int_{S:\, C=\partial\Sigma} dS^{\mu\nu} F_{\mu\nu}[\mathcal{V}] \right),$$

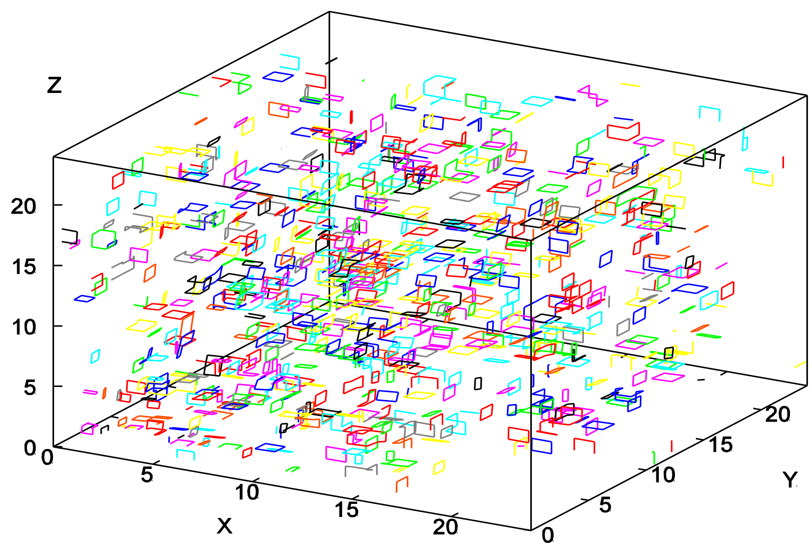
(no path ordering), and the decomposed $V_{x,\mu}$ corresponds to the Lie algebra value of $\mathcal{V}_{x,\mu}$ and the field strength on a lattice is given by plaquet of $V_{x,\mu}$

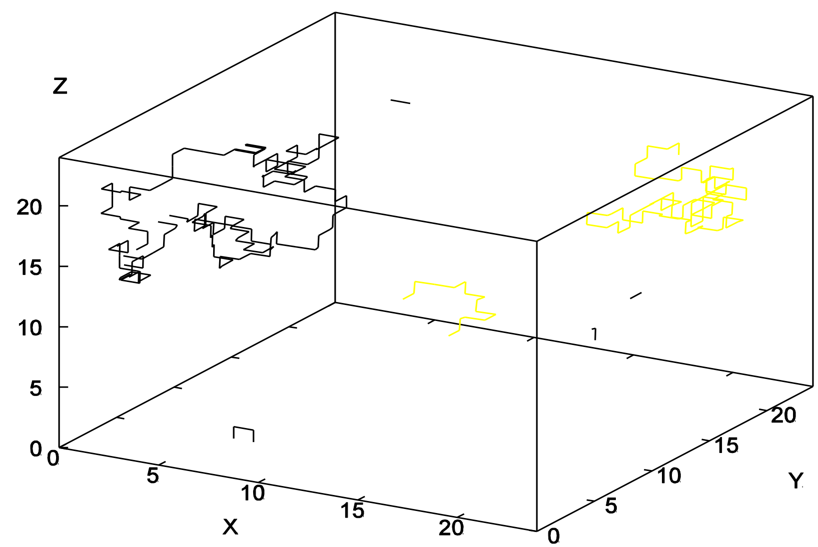
Non-Abelian magnetic monopole loops: 24^4 laiitce β=6.0

Projected view (x,y,z,t) ➔(x,y,z)

(left lower) loop length 1-10
(right upper) loop length 10 -- 100
(right lower) loop length 100 -- 1000





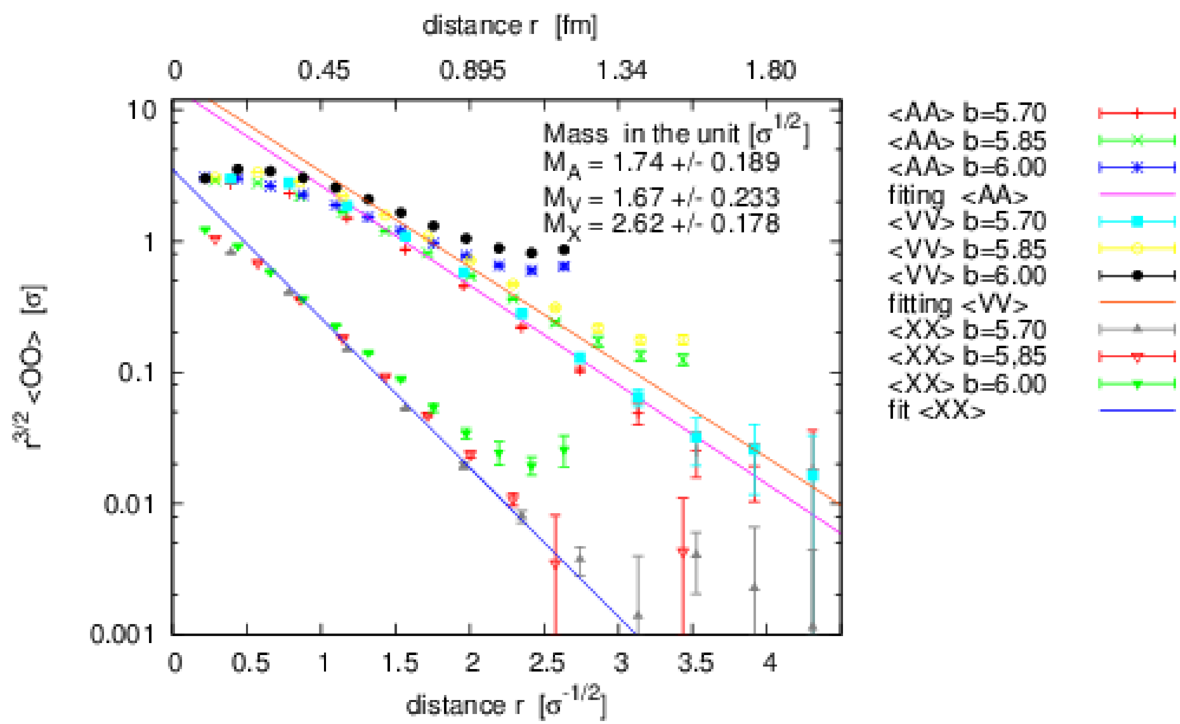


The gauge boson propagator $D^{XX}_{\mu\nu}(x-y)$ is related to the Fourier transform of the massive propagator

$$D^{XX}_{\mu\nu}(x-y) = \langle X_\mu(x)X_\nu(y)\rangle = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} D^{XX}_{\mu\nu}(k)$$

The scalar type of propagator as function r should behave for large M_x as

$$D^{XX}(r) = \langle X_\mu(x)X_\mu(y)\rangle = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{3}{k^2+M_X^2} \simeq \frac{3\sqrt{M}}{2(2\pi)^{3/2}} \frac{e^{-M_x r}}{r^{3/2}}$$

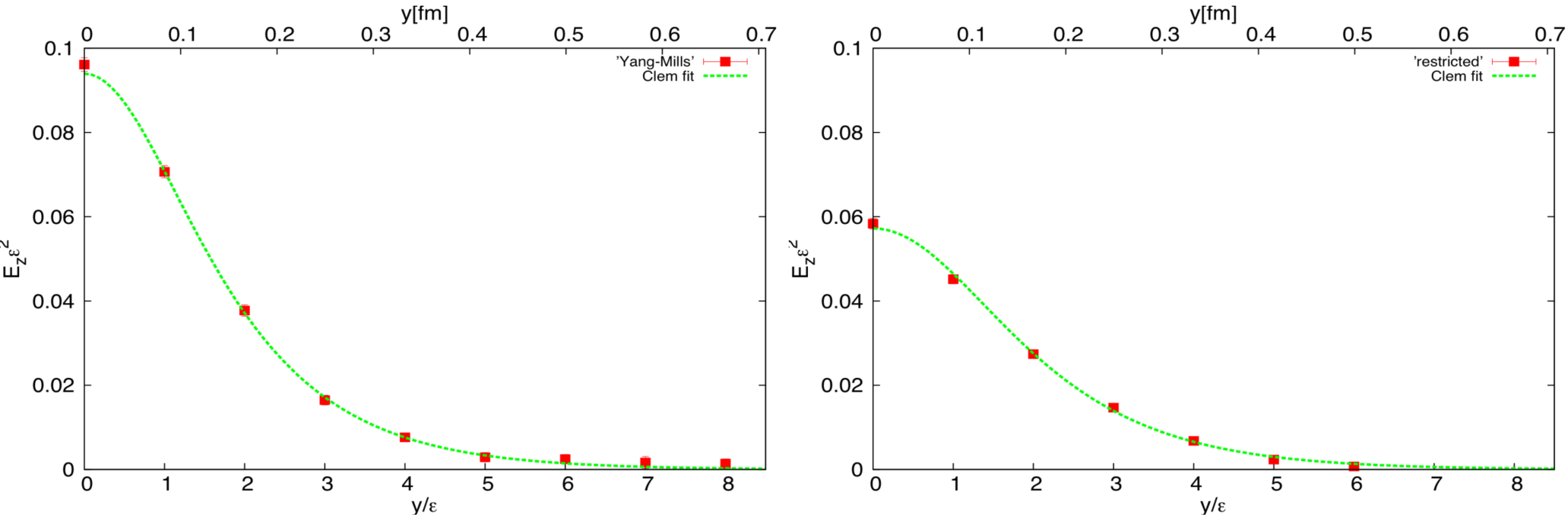


results of fitting

Fitting function:

$$E_x(y) = aK_0[\sqrt{\mu^2 y^2 + \alpha^2}], \quad a = \frac{\phi}{2\pi}\frac{\mu^2}{\alpha}/K_1[\alpha].$$

Original Yang-Mills field restricted field



	a	μ	α
link field U	0.341(0.167)	0.781(0.087)	1.308(0.393)
restricted field V	0.368(0.249)	0.782(0.109)	1.748(0.548)

■ GL parameter and type of dual superconductor

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